

Problems in Topology (Math436)

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Read Chapter 1.

- (1) Let X and Y be sets. Show that the following statements are equivalent:
- $|X| = |Y|$
 - There are maps $f : X \rightarrow Y$ and $g : Y \rightarrow X$, which are both 1-1.
 - There is a map $f : X \rightarrow Y$ which is 1-1 and a map $g : X \rightarrow Y$ which is onto.
 - There are maps $f : X \rightarrow Y$ and $g : Y \rightarrow X$, which are both onto.

Hint: You may use the following *Axiom of Choice* (which is equivalent to the Lemma of Zorn): If $(X_\alpha : \alpha \in A)$ is a family of **non empty** sets indexed by the set A , then there is a function $h : A \rightarrow \bigcup_{\alpha \in A} X_\alpha$ so that $h(\alpha) \in X_\alpha$ for all $\alpha \in A$. (i.e. h chooses an element out of each X_α).

- (2) Proof Theorem 1.15 from text.
(3) Show that

$$\{F : F \subset \mathbb{N}, F \text{ finite} \}$$

is countable.

Hint: Cantor-Schröder-Bernstein makes life much easier!

- (4) (*) Let X be an infinite set and $A \subset X$ finite. Show that X and $X \setminus A$ are equipotent.

Hint: Show the claim first for an infinite and countable set.