Problems to Introduction to Real Analysis, (Math446)

Due: 09/16/02

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Problem 1. Every finite subset of $\mathbb{R}$ is closed.

Problem 2. Find a sequence of open sets (U_n) and sequence of closed sets (V_n) so that

$$\bigcap_{n\in\mathbb{N}} U_n \text{ is not open,}$$
$$\bigcup_{n\in\mathbb{N}} V_n \text{ is not closed.}$$

Problem 3. Using the "finite covering property" show that the set

$$\{0\} \cup \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$$

is compact and that the set

$$\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$$

is not compact.

Problem 4. Let $A \subset \mathbb{R}$. We call the set

$$\overline{A} = \bigcap\{B : A \subset B \subset \mathbb{R}, B \text{ closed }\},$$

the *closure of* A. and the set

$$A^\circ = \bigcup\{U : U \subset A, U \text{ open}\},$$

the *open interior of* A. *Show that*

a) $\overline{A}$ *is closed and that* A *is closed if and only if* $\overline{A} = A$.

b) A° *is open that* A *is open if and only if* $A^\circ = A$.

Problem 5. *Determine (with proof) the closure and the interior of the following sets:* $[0,1) \cup \{2\}$, $\mathbb{N}$, $\mathbb{Q}$.

Show that for $A, B \subset \mathbb{R}$

$$\overline{A \cap B} \subset \overline{A} \cup \overline{B},$$

and that equality does not necessarily hold.

Problem 5. *Let* $A \subset \mathbb{R}$ *and* $a \in \mathbb{R}$. *Then* a *is a limit point of* A *if and only if there is sequence* $(a_n) \subset A$ *which converges to* a.

Problem 6. *Let* $f : \mathbb{R} \to \mathbb{R}$ *be contiunous. Show that the image of a comapct set* $K \subset \mathbb{R}$ *under* f *(i.e.,* $f(K) = \{f(x) : x \in K\}$*) is compact.*

Show by examples that the image of open/closed sets under continuous maps $f : \mathbb{R} \to \mathbb{R}$ *do not need to be open/closed.*