HOMEWORK 2

Exercise 9 Pg. 36

$P \Rightarrow Q$ is true in all cases except when P is true and Q is false. But when P is true and Q is false, $Q \Rightarrow P$ is true. Hence $(P \Rightarrow Q) \vee (Q \Rightarrow P)$ is true in all cases.

Exercise 6 Pg 44

(a) Because n and m are even, we can write $n = 2t$ and $m = 2s$ for some $t, s \in \mathbb{Z}$. Then $n + m = 2(t+s)$. Therefore $n+m$ is even.

(b) If $n+m$ is odd, then either n is odd or m is odd.

(c) CONVERSE: "If $n+m$ is even, then n and m are even."

This is FALSE: counter Example →
3 and 5 are odd, but $3+5$ is even.

Exercise 20 Pg 45

(a) We first prove that if n is even, then n^3 is even. Since n is even we can write $n = 2t$ for $t \in \mathbb{Z}$. Then $n^3 = (2t)^3 = 8t^3 = 2(4t^3)$ where $4t^3 \in \mathbb{Z}$. $\Rightarrow$ n^3 is even.

We now will prove the converse, namely, if n^3 is even, then n is even. We will do this by proving the contrapositive of the converse, which is the statement: if n is odd, then n^3 is odd. So we suppose that n is odd, so $n = 2t + 1$ for $t \in \mathbb{Z}$. We get $n^3 = (2t+1)^3 = 2(4t^3 + 6t^2 + 3t) + 1$, where $4t^3 + 6t^2 + 3t \in \mathbb{Z}$. Hence n^3 is an odd integer.

(b) $P =$ "n is odd" $\longrightarrow$ $\neg P =$ "n is even"

$Q =$ "n^3 is odd" $\longrightarrow$ $\neg Q =$ "n^3 is even"

Now "$\neg P \Leftrightarrow \neg Q$" is true by part (a). From exercise 11(b), it follows that $P \Leftrightarrow Q$ is true.

Exercise 12 Pg 59

(a) Let $n \in A$. Then $n = 4m$ for $m \in \mathbb{Z}$. Hence $n^2 = 16m^2 = 4(4m^2)$. ~~where~~ Thus $n \in B$. Therefore, $A \subseteq B$.

(b) B is not a subset of A because 2 is an element of B but 2 is not an element of A.