Problems in Topology (Math436)

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Read Chapter 2.

(1) Verify that d_1 and d_∞ in Example 2.9 are metrics.

(2) Prove that for a metric space (X, d) and $A \subset X$

$$A \text{ open} \iff \exists (x_i)_{i\in I} \subset X \, \exists (\varepsilon_i)_{i\in I} \subset (0,1) \quad A = \bigcup_{i\in I} B(x_i, \varepsilon_i).$$

(3) Let V be a vector space over $\mathbb{R}$. A function'

$$\|\cdot\| : V \to [0, \infty), \quad v \mapsto \|v\|$$

is called a *norm* if

(i) $\|x\| = 0$ if and only if $x = 0$

(ii) $\|a \cdot x\| = |\alpha| \|x\|$, if $\alpha \in \mathbb{R}$ and $x \in V$.

(iii) $\|x + y\| \le \|x\| + \|y\|$, if $x, y \in V$.

Define for $x, y \in V$ $d(x, y) = \|x - y\|$ and show that d is a metric.

(4) Problem 12 page 31.

(5) (*) For a particular signal processing problem Engineer A defines a function $d : X \times X \to \mathbb{R}$ with the following properties

(i) $d(x, y) = 0$ if and only if $x = y$

(ii) $d(x, y) = d(y, x)$

(iii) $d(x, z) \le d(x, y) + d(y, z)$

Engineer B points out that d should satisfy one more condition. Is he right? If so, which is the condition?

Engineer A asserts that d is still a metric. Why?