HOMEWORK #3

SECTION 2.2 → #7:

(a) First prove that $A \cup B = (a,d)$. Suppose that $x \in A \cup B$. Then $a < x < b$ or $c < x < d$. In either case, using the above inequalities we have $a < x < d$, so $x \in (a,d)$. Thus $A \cup B \subseteq (a,d)$. Conversely, suppose $x \in (a,d)$. Then $a < x < d$. If $x < b$, then $x \in A$. If $x > b$ then $c < x < d$ so $x \in B$. Thus $x \in A \cup B$. It follows that $(a,d) \subseteq A \cup B$ and therefore $A \cup B = (a,d)$ an open interval.

(b) We now prove that $A \cap B = (c,b)$. Let $x \in A \cap B$. Then $x \in A \cap B$. Then $a < x < b$ and $c < x < d$. Therefore, $x > c$ and $x < b$, so $x \in (c,b)$. So, $A \cap B \subseteq (c,b)$. Let $x \in (c,b)$. Then $c < x < b$. Since $a < c < b < d$, we have $a < x < b$ and $c < x < d$. Therefore $x \in A \cap B$. and we have $(c,b) \subseteq A \cap B$. We conclude that $A \cap B = (c,b)$.

SECTION 2.2 → #15:

(a) We want to prove that if A is a subset of $B \cap C$, then A is a subset of B and A is a subset of C. So we start by assuming that A is a subset of $B \cap C$. Let $x \in A$. Then $x \in B \cap C$ by our assumption that A is a subset of $B \cap C$. So $x \in B$ and $x \in C$. So any element of A is an element of B and any element of A is an element of C. Thic proves that $A \subseteq B$ and $A \subseteq C$.

(b) CONTRAPOSITIVE:

If A is NOT a subset of B or A is not a subset of C, then A is NOT a subset of $B \cap C$.

(c) CONVERSE:

If A is a subset of B and A is a subset of C, then A is a subset of $B \cap C$.

⇒ TRUE

To prove this we start by assuming that A is a subset of B and A is a subset of C. Now let $x \in A$. Then $x \in B$ since A is a subset of B and $x \in C$ since A is a subset of C. Therefore, $x \in B \cap C$. Thus $A \subseteq B \cap C$.

SECTION 2.3 → #5:

(a) False
(b) True
(c) True
(d) True
(e) True
(f) False
(g) True

SECTION 2.3 → #12:

We claim that $\bigcup_{i=2}^{\infty} A_i = (0,\infty)$. Since each $A_i \subseteq (0,\infty)$, $\bigcup_{i=2}^{\infty} A_i \subseteq (0,\infty)$ by exercise 9. Let $x \in (0,\infty)$. By exercise 13 of section 1.1, there exists an integer n s.t $n^{-1} < x < n$. Therefore, $x \in A_n$. Hence $x \in \bigcup_{i=2}^{\infty} A_i$ and we get $(0,\infty) \subseteq \bigcup_{i=2}^{\infty} A_i$. It follows that $\bigcup_{i=2}^{\infty} A_i = (0,\infty)$.

We prove that $\bigcap_{i=1}^{\infty} A_i = (\frac{1}{2}, 2]$. First $\bigcap_{i=1}^{\infty} A_i \subseteq A_2 = (\frac{1}{2}, 2]$. Now let $x \in (\frac{1}{2}, 2]$. Then $\frac{1}{2} < x \leq 2$. If $n > 2$, we get $1/n \leq 1/2 < x \leq 2 \leq n$, so $x \in A_n$ for all n. Therefore, $(\frac{1}{2}, 2] \subseteq \bigcap_{i=1}^{\infty} A_i$. We conclude $\bigcap_{i=1}^{\infty} A_i = (\frac{1}{2}, 2]$.