Read Chapter 2 (finish)

1. Let \((X, d)\) be a metric space. For \(A, B \subset X\) show that
   \[\overline{A \cup B} = \overline{A} \cup \overline{B}\]
   and \(\overline{A \cap B} \subset \overline{A} \cap \overline{B}\)
   and find an example for which \(\overline{A \cap B} \neq \overline{A \cap B}\).

2. For \(x, y \in \mathbb{R}\) define
   \[\rho(x, y) = |\arctan(x) - \arctan(y)|\]
   a) Show that \(\rho\) is a metric on \(\mathbb{R}\) which has the same open sets
   as the usual metric.
   b) Show that \((\mathbb{R}, \rho)\) is not complete.

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4. Give an example of a metric space \((X, d)\) and closed sets \(F_n \subset X, n \in \mathbb{N}\),
   for which \(\bigcup F_n\) is not closed.

5. (*) Find a metric \(\rho\) on \((0, 1)\) which has the same open sets as
   for the usual metric, but for which \(((0, 1), \rho)\) is complete.