

## Problems in Topology (Math436)

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Read Chapter 2 (finish)

- (1) Let  $(X, d)$  be a metric space. For  $A, B \subset X$  show that

$$\overline{A \cup B} = \overline{A} \cup \overline{B} \text{ and } \overline{A \cap B} \subset \overline{A} \cap \overline{B}$$

and find an example for which  $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$ .

- (2) For  $x, y \in \mathbb{R}$  define

$$\rho(x, y) = |\arctan(x) - \arctan(y)|$$

- a) Show that  $\rho$  is a metric on  $\mathbb{R}$  which has the same open sets as the usual metric.

- b) Show that  $(\mathbb{R}, \rho)$  is not complete.

- (3) Problem 13, page 31

- (4) Give an example of a metric space  $(X, d)$  and closed sets  $F_n \subset X$ ,  $n \in \mathbb{N}$ , for which  $\bigcup F_n$  is not closed.

- (5) (\*) Find a metric  $\rho$  on  $(0, 1)$  which has the same open sets as for the usual metric, but for which  $((0, 1), \rho)$  is complete.