Problems to Introduction to Real Analysis, (Math447)

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Problem 1. Let X be a normed vector space, we denote the norm by $\|\cdot\|$. Show the following are equivalent,

a) X is complete.

b) For every sequence (x_n) in X for which $\sum_{n\in\mathbb{N}} \|x_n\| < \infty$ the series $\sum x_n$ converges in X.

[Hint: You can use that, if a Cauchy (x_n) sequence has a convergent subsequence it converges.]

Problem 2. Let

$$C^1[0,1] = \{f : [0,1] \to \mathbb{R} : f \text{ is continuously differentiable}\}.$$

and put for $f \in C^1[0,1]$, $||f|| = |f(0)| + \max_{t\in[0,1]} |f'(t)|$. Show that $\|\cdot\|$ is a norm on $C^1[0,1]$ and that $C^1[0,1]$ is complete with that norm.

Problem 3. Now consider on $C^1[0,1]$ the "wrong norm", namely the sup norm, i.e.$||f|| = \max_{t\in[0,1]} |f(t)|$. Show that $C^1[0,1]$ with that norm is not complete.

Problem 4. Problem 4, page 165.

Problem 5. Problem 6, page 166.

Problem 6. Problem 7, page 166.