Problems in Topology (Math436)
Prof.: Thomas Schlumprecht Due: 2/14/2008

Read Chapter 3.

(1) Problem 3 Page 40.
(2) Problem 4 Page 40.
(3) Problem 9 Page 40.
(4) (*) Consider the vector space c_0 of all sequences in $\mathbb{R}$ which converge to 0 (note, without writing down a proof, that this is indeed a vector space over $\mathbb{R}$). For $\overline{x} = (x_n : n \in \mathbb{N}) \in c_0$ we define
$$\|\overline{x}\| = \max\{|x_n| : n \in \mathbb{N}\}.$$
Show that $\|\cdot\|$ is a norm on c_0.
Let $d(\cdot,\cdot)$ be the metric induced by $\|\cdot\|$ (i.e. $d(x,y) = \|x-y\|$).
Show that (c_0, d) is a complete metric space.