HOMEWORK 5

#2b, Pg 134

NOT ASSOCIATIVE

Counter Example:

$$\left.\begin{aligned} (2*3)*4 &= 12*4 = 54 \\ 2*(3*4) &= 2*18 = 42 \end{aligned}\right\} \text{not equal}$$

COMMUTATIVE

Proof:

Let $n, m \in \mathbb{Z}$. Then $n*m = 6 + nm = 6 + mn = m*n$.

#2c, Pg 134

NOT ASSOCIATIVE

Counter Example:

$$\left.\begin{aligned} (2*3)*4 &= 36*4 = 20736 \\ 2*(3*4) &= 2*144 = 82944 \end{aligned}\right\} \text{not equal}$$

COMMUTATIVE

Proof:

Let $m, n \in \mathbb{Z}$. Then $n*m = n^2m^2 = m^2n^2 = m*n$

#3 Pg 134

(a) Suppose $|A| \geq 2$, Let a and b be two distinct elements of A. Let f, g be elements of $F(A)$ s.t. $f(a) = b$, $f(b) = a$, $g(a) = a$, and $g(b) = a$. Then $(fg)(a) = f(g(a)) = f(a) = b$ and $(gf)(a) = g(f(a)) = g(b) = a$. Thus $fg \neq gf$ and so composition on $F(A)$ is NOT commutative.

(b) Suppose $|A| = 1$, say $A = \{a\}$. Then $F(A)$ has only one element; namely, the identity mapping i_A. Therefore $F(A)$ must be commutative.

#12 Pg 135

Suppose a has another inverse b. Then $a * b = b * a = e$. Using the fact that $*$ is associative, we can write $b = b * e = b * (a * a^{-1}) = (b * a) * a^{-1} = e * a^{-1} = a^{-1}$. Thus $b = a^{-1}$, so a^{-1} is unique.

#16 Pg 135

(a) Let $a, b \in E$. Then $a = 2x$ and $b = 2y$ for some $x, y \in \mathbb{Z}$. Then $ab = (2x)(2y) = 2(2xy) \in E$. Therefore E is closed under multiplication.

(b) Let $a, b \in O$. Then $a = 2x+1$ and $b = 2y+1$ for some $x, y \in \mathbb{Z}$. Then $ab = (2x+1)(2y+1) = 4xy + 2x + 2y + 1$ $= 2(2xy + x + y) + 1 \in O$. Therefore O is closed under multiplication.