Problems in Topology (Math436)
Prof.: Thomas Schlumprecht   Due: 2/21/2008

Read Chapter 4.

(1) Problem 2, page 52.

(2) Problem 4, page 52.

(3) Let $(X, \mathcal{T})$ be a topological space. We say that a sequence $(x_n) \subset X$ converges to x with resepct to $\mathcal{T}$, if for every neighborhood A of x there is an $N \in \mathbb{N}$ so that $x_n \in A$ for all $n \in \mathbb{N}$.
   Show that if $A \subset X$ and $(x_n) \subset A$ converges to some $x \in X$ then $x \in \overline{A}$.
   Note: We will see later that the converse (i.e. every element of $\overline{A}$ is the limit of some sequence in A), is not true for general topological spaces.

(4) The *Sorgenfrey line*
   (a) Show that $\mathcal{B} = \{[a, b) : a < b\}$ is a basis of a topology on $\mathbb{R}$. We denote this Topology by $\mathcal{S}$.
   (b) $\mathcal{T}_u \subset \mathcal{S}$, but $\mathcal{T}_u \neq \mathcal{S}$, where $\mathcal{T}_u$ denotes the usual topology on $\mathbb{R}$.
   (c) Find a sequence in $\mathbb{R}$ which converges in the usual topology, but does not converge in $\mathcal{S}$.

(5) (*) Let $\mathcal{S}$ be the topology introduced in the previous example.
   (a) Show that $\overline{\mathbb{Q}} = \mathbb{R}$ with respect to $\mathcal{S}$.
   (b) Show that $\mathcal{S}$ is not second countable.
   (c) Show that $\mathcal{S}$ Lindelöf.
   (d) Deduce that the topology $\mathcal{S}$ is not generated by a metric.