HOMEWORK #6

Problem #1 Pg 157

(a) Since $a+(-a) = -a + a = 0$, it follows that a is the additive inverse of $(-a)$; in other words $-(-a) = a$.

(b)
$$\begin{aligned} (-a)(-b) &= -(a(-b)) && \text{by P3 of Prop. 5.1.1} \\ &= -(-(ab)) && \text{by P3 of Prop. 5.1.1} \\ &= ab && \text{by P4 of Prop. 5.1.1} \end{aligned}$$

(c)
$$\begin{aligned} a(b-c) &= a(b+(-c)) = ab + a(-c) && \text{by A8} \\ &= ab + (-(ac)) && \text{by P3} \\ &= ab - ac \end{aligned}$$

Problem #2 Pg 157

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$$\begin{aligned} -(a+b) &= (-1)(a+b) && \text{by P7} \\ &= (-1)a + (-1)b && \text{by A8} \\ &= -a + (-b) && \text{by P7} \\ &= -a - b \end{aligned}$$

Problem #5 Pg 157

(a) Suppose that $a<0$ and $b<0$. Then $-a>0$ and $-b>0$ by Q2. Hence $(-a)(-b)>0$ by Q3. But, by P5, $(-a)(-b) = ab$. Therefore, $ab>0$.

(b) Suppose $a<b$ and $b<c$. Then $b-a \in \mathbb{Z}^+$ and $c-b \in \mathbb{Z}^+$. So by A9, $c-a = (c-b)+(b-a) \in \mathbb{Z}^+$. Thus $a<c$.

(c) Suppose that $a<b$. Then $b-a \in \mathbb{Z}^+$. Therefore, $(b+c)-(a+c) = b-a \in \mathbb{Z}^+$. Thus $a+c < b+c$.

(d) Suppose $a<b$ and $c>0$. Then $b-a \in \mathbb{Z}^+$ and $c \in \mathbb{Z}^+$. By A9 $(b-a)c \in \mathbb{Z}^+$. But $(b-a)c = bc-ac$ by P6. Therefore, $bc-ac \in \mathbb{Z}^+$, so that $ac<bc$.

(e) Suppose $a<b$ and $c<0$. Then $b-a>0$ and $c<0$. So $a-b = -(b-a)<0$. By Q5, $(a-b)c>0$. By P6, $ac-bc = (a-b)c>0$. It follows that $ac>bc$.

Problem #16, Pg. 158

Let $x,y,z \in \mathbb{Z}$. Suppose that $xy = yz$ and $x>0$. Then $x(y-z) = xy-yz = 0$. Since $x>0$, it follows from exercise 15 that $y-z=0$ and therefore $y=z$.