Problems to Introduction to Real Analysis, (Math446)

Due: 11/23/04

Problem 1. Let $(X, \mathcal{T})$ be a topological space. $A, B \subset X$

a) $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$,

b) $\overline{A} \setminus \overline{B} \subset \overline{A \setminus B}$.

Problem 2. Consider on $\mathbb{R}$ the topology $\mathcal{T}_h$ generate by all half open intervals of the form $[a, b[$. Denote the usual topology on $\mathbb{R}$ by $\mathcal{T}_u$.

a) Show that $\mathcal{T}_h$ at least as fine as $\mathcal{T}_u$ (i.e. show that $\mathcal{T}_u \subset \mathcal{T}_h$.

b) Show that the intervals $[a, b[$ are also closed with respect to $\mathcal{T}_h$.

c) Does $(\frac{1}{n})$ converge in $\mathcal{T}_h$? Does $(-\frac{1}{n})$ converge in $\mathcal{T}_h$? Support your answers.

d) Show that $\mathbb{R}$ is separable with respect to $\mathcal{T}_h$.

Problem 3. Let $\mathcal{B}$ be a base for $(X, \mathcal{T})$ and let $\mathcal{B} \subset \mathcal{B}^* \subset \mathcal{T}$. Show that $\mathcal{B}^*$ is also a base for $\mathcal{T}$.

Problem 4. The topology generated by a metric space which is separable has a base which is countable.

Problem 5. Consider the set of all half planes on $\mathbb{R}^2$, i.e.

$$\mathcal{B} = \bigcup_{a \in \mathbb{R}} \big\{ \{(x, y) : y > a\}, \{(x, y) : y < a\}, \{(x, y) : x > a\}, \{(x, y) : x < a\} \big\}.$$

Show that $\mathcal{B}$ is a subbasis which generates the usual topology.

Problem 6.* There is no metric d on $\mathbb{R}$ so that the topology generated by d equals to the topology $\mathcal{T}_h$ described in Problem 2.
Hint: Use Problems 2 d and Problem 4.