HOMEWORK #7 SOLUTIONS

Problem #1c – Pg 169

Let $P(n)$ be the statement: $1^3+2^3+\ldots+n^3 = n^2(n+1)^2/4$.
Since $1^3 = 1^2(1+1)^2/4$, $P(1)$ is true. Suppose that $P(k)$ is true. Then
$1^3+2^3+\ldots+k^3 = k^2(k+1)^2/4$. Hence,

$$\begin{aligned}1^3+2^3+\ldots+k^3+(k+1)^3 &= k^2(k+1)^2/4 + (k+1)^3\\ &= ((k+1)^2/4)(k^2+4k+4)\\ &= (k+1)^2(k+2)^2/4.\end{aligned}$$

This proves that $P(k+1)$ is true. Therefore $P(n)$ is true for all positive integers n.

Problem #2b – Pg 169

Let $P(n)$ be the statement:
$2^2+4^2+6^2+\ldots+(2n)^2 = (2n)(2n+1)(2n+2)/6$. Then $P(1)$ is true, since
$2^2 = (2)(2+1)(2+2)/6$. Now suppose that $P(k)$ is true. Then
$2^2+4^2+\ldots+(2k)^2 = (2k)(2k+1)(2k+2)/6$ and we get:

$$\begin{aligned}2^2+\ldots+(2k)^2+(2k+2)^2 &= ((2k)(2k+1)(2k+2)/6) + (2k+2)^2\\ &= ((2k+2)/6)(4k^2+2k+12k+12)\\ &= (2k+2)(2k+3)(2k+4)/6.\end{aligned}$$

This proves that $P(k+1)$ is true. Hence, by induction is true for all positive integers n.

Problem #3 – Pg 169

Let $P(n)$ be the statement: $1+a+a^2+\ldots+a^n = (a^{n+1}-1)/(a-1)$.
$P(1)$ is true because $1+a = (a^2-1)/(a-1)$. If $P(k)$ is true, that is, if $1+a+a^2+\ldots+a^k = (a^{k+1}-1)/(a-1)$, then

$$\begin{aligned}1+a+a^2+\ldots+a^k+a^{k+1} &= ((a^{k+1}-1)/(a-1)) + a^{k+1}\\ &= \frac{a^{k+1}-1+a^{k+1}(a-1)}{a-1}\\ &= \frac{a^{k+2}-1}{a-1}\end{aligned}$$

Problem #9 - Pg 169

(a) $H_2=1$, $H_3=3$, $H_4=6$, $H_5=10$, $H_6=15$

(b) $H_{n+1}=n+H_n$

(c) $H_n=n(n-1)/2$

Problem #26 - Pg 171

Apply the binomial theorem with $a=1$, $b=-1$.