Problems in Real Analysis II (Math608) **Due: 4/14/10**

Problem 1.

a) Show that the closure of each convex set in a topological vector space is convex.
b) Show that in $\mathbb{R}^n$ each internal point of a convex set is an interior point.
c) Give an example of a subset of $\mathbb{R}^2$ which has an internal point which is not in the interior of that set.

Problem 2. Show that $B_{L_1[0,1]}$ does not have any extreme points. Deduce from this that $L_{[0,1]}$ is not isometric to the dual of a Banach space.

Problem 3. Determine the extreme points of the unit balls of c_0 and ℓ_1 and ℓ_∞. Deduce that c_0 is not the dual of a some Banach space X.

Problem 4. Determine the extreme points of the unit ball of $C[0,1]$.
Hint: The answer for $\mathcal{F} = \mathbb{R}$ and $\mathbb{F} = \mathbb{C}$ will be (very) different.

Problem 5. Let (f_n) be a sequence of continuous functions on $[0,1]$ such that, for each $x \in [0,1]$ there is an $n(x)$ such that $f_n(x) \geq 0$ for all $n \geq n(x)$.

Prove that there is an open, not empty interval $I \subset [0,1]$ and an $N \in \mathbb{N}$ so that $f_n(x) \geq 0$ for all $x \in I$ and $n \geq N$.

Problem 6. Let X be a Banach space and Y a closed subspace of X. Show that every extreme point of the unit ball of the dual Y^* extends to an extreme point of the unit ball of X^*.

Problem 7. Only using the definition of topological vector spaces prove that a linear function on a topological vector space is continuous if it is bounded on some open neighborhood of 0.