1a/ page 63 $\lim_{x \to 2} x^2 - x + 1 = 3$

Let $\varepsilon > 0$ choose $\delta = \text{Min}\left(\frac{\varepsilon}{3}, 1\right)$

Then if $|x-2| < \delta$

we have $|x^2 - x + 1 - 3|$

$= |x^2 - x - 2|$

$= \underbrace{|x-2|}_{<\delta} \cdot |x+1|$

$\leq |x-2| \cdot |2+1| = 3|x-2| < 3\frac{\varepsilon}{3} = \varepsilon$

1b / page 64 $\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$

Let $\varepsilon > 0$ Choose $\delta = \varepsilon$

Then if $0 < |x-1| < \delta$

$$\left|\frac{x^2-1}{x-1} - 2\right| = \left|\frac{(x+1)(x-1)}{x-1} - 2\right|$$

$$= |x+1-2| = |x-1| < \varepsilon$$

3 (c) / page 64 $\lim_{x \to \sqrt{\pi}} \frac{\sqrt[3]{\pi - x^2}}{x + \pi}$

$(*)$ $\lim_{x \to \sqrt{\pi}} (\pi - x^2) = \lim_{x \to \sqrt{\pi}} \pi - \left(\lim_{x \to \sqrt{\pi}} x\right)^2$

(Summation & Product rule)

$= \pi - \pi = 0$

$(**)$ $\lim_{x \to \sqrt{\pi}} x + \pi = \lim_{x \to \sqrt{\pi}} x + \pi = \sqrt{\pi} + \pi$

Since $\lim_{x\to 0} \sqrt[3]{x} = 0$ (*) implies

that $\lim_{x\to\sqrt{\pi}} \sqrt[3]{\pi - x^2} = 0$

and thus by quotient rule

$$\lim_{x\to\sqrt{\pi}} \frac{\sqrt[3]{\pi - x^2}}{x+\pi} = 0$$

3(e) / page 64

$$-|x| \leq x \sin\frac{1}{x} \leq |x|$$

Thus since $\lim_{x\to 0} |x| = 0$

by Squeeze Theorem $\lim_{x\to 0} x\sin\frac{1}{x} = 0$

Theorem 3.8 on sums direct proof

Assume $\lim\limits_{x \to a} f(x) = L$ $\lim\limits_{x \to a} g(x) = K$

to show: $\lim\limits_{x \to a} f(x) + g(x) = L + K$

Proof:

Let $\varepsilon > 0$ we find

$\delta_1 > 0$ so that $|x - a| < \delta_1 \Rightarrow |f(x) - L| < \frac{\varepsilon}{2}$

$\delta_2 > 0$ so that $|x - a| < \delta_2 \Rightarrow |g(x) - K| < \frac{\varepsilon}{2}$

Choose $\delta = \min(\delta_1, \delta_2)$

then, if $|x - a| < \delta$,

$|f(x) + g(x) - (L + K)| = |(f(x) - L) + (g(x) - K)|$

$\leq |f(x) - L| + |g(x)\ \ K| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$

5 / page 69 (answer in book is of course wrong !)

$$\sin(x+3) = \sin x \cdot \cos 3 + \cos x \cdot \sin 3$$

We write

$$\frac{\sin(x+3) - \sin(3)}{x} = \frac{\sin x \cdot \cos 3 + \cos x \sin 3 - \sin 3}{x}$$

$$= \frac{\sin x}{x} \cos 3 + \sin 3 \, \frac{\cos x - 1}{x}$$

$$= \frac{\sin x}{x} \cos 3 + \sin 3 \, \frac{(\cos x - 1)(\cos x + 1)}{x(\cos x + 1)}$$

$$= \frac{\sin x}{x} \cos 3 + \sin 3 \, \frac{\cos^2 x - 1}{x(\cos x + 1)}$$

$$= \frac{\sin x}{x} \cos 3 - \sin 3 \, \frac{\sin^2}{x} \, \frac{1}{\cos x + 1}$$

$$= \underset{\downarrow \atop 1}{\frac{\sin x}{x}} \cos 3 - (\sin 3)\underset{\downarrow \atop 0}{(\sin x)}\underset{\downarrow \atop 1}{\left(\frac{\sin x}{x}\right)} \underset{\downarrow \atop 2}{\frac{1}{\cos x + 1}}$$

Thus $\displaystyle \lim_{x \to 0} \frac{\sin(x+3) - \sin 3}{x} = \cos 3$

7 | page 71

(a) let $M > 0$
then there exists $\delta > 0$
So that:
If $|x-a| < \delta$ then $g(x) \geq M$

Since $f(x) \geq g(x)$ also
If $|x-a| < \delta$ then $f(x) \geq M$

(b) (Squeeze Theorem at ∞)

assume $f(x) \leq g(x) \leq h(x)$ &
$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} h(x) = L$

Let $\varepsilon > 0$ then there is an
$M_1 > 0$ so that
if $x > M_1$ then $|f(x) - L| < \varepsilon$
there is an
$M_2 > 0$ so that
if $x > M_2$ then $|h(x) - L| < \varepsilon$

Choose $M = \operatorname{Max}(M_1, M_2)$

Then if $|x - \cancel{a}| < \delta$ $g(x) - L \leq h(x) - L < \varepsilon$
and $g(x) - L \geq f(x) - L > -\varepsilon$
Thus $|g(x) - L| < \varepsilon$