Practice Test on Sets and Functions (Chapter 2 and 3), Math220

(1) Let A and B be sets and let $(A_i : i \in I)$ be a family of sets. Define the following:
 (a) The intersection of A and B,
 (b) the union of A and B,
 (c) the intersection of $(A_i : i \in I)$,
 (d) the union of $(A_i : i \in I)$,
 (e) the Cartesian product of A and B,
 (f) the difference of A and B.

(2) For a set A define the power set $P(A)$.

(3) Let $f : A \to B$ be function from A to B, define what it means, that
 (a) f is injective,
 (b) f is surjective,
 (c) f is bijective,
 (d) f is invertible.

(4) Find (with proof) $f^{-1}(X)$, and determine whether f is injective or surjective, where f and X are the following

(5) $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = 2n$, and $X = 4\mathbb{Z}$,

(6) $f : \mathbb{Z} \to \mathbb{N}$, $f(n) = n^2$, and $X = \{2^n : n \in \mathbb{N}\}$,

(7) $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = n^2$, and $X = -\mathbb{N}$,

(8) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 - x - 2$ and $X = -\{0\}$,

(9) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 - x - 2$ and $X = \mathbb{R}^+$,

(10) Let $f : A \to B$ be function from A to B and let $X \subseteq A$, and $Y \subseteq B$, define
 (a) $f^{-1}(Y)$,
 (b) $f(X)$.

(11) For two sets A and B prove:

$$A \subset B \iff A = A \cap B.$$

$$A \subset B \iff B = A \cup B.$$

(12) Write down $P(A)$ for:

$$A = \{1, 2\},\ A = \emptyset,\ A = \{\emptyset\}.$$

(13) Assume that $(A_i : i \in I)$ is a family of subsets of a set X (complements are taken in X). Prove

$$\overline{\bigcup_{i\in I} A_i} = \bigcap_{i\in I} \overline{A_i} \text{ and } \overline{\bigcap_{i\in I} A_i} = \bigcup_{i\in I} \overline{A_i}$$

(14) Let $f : A \to B$, and X, Y be subsets of B. Prove:
 (a) $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$,

(b) $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$,
(c) If $X \subseteq Y$ then $f^{-1}(X) \subseteq f^{-1}(Y)$,
(d) $f(f^{-1}(X)) \subseteq X$. Find an example for which we have $f(f^{-1}(X)) \subsetneq X$.

(15) Let $f : A \to B$, and X, Y be subsets of A. Prove:
(a) $f(X \cap Y) \subseteq f(X) \cap f(Y)$. Find an example for which we have $f(X \cap Y) \subsetneq f(X) \cap f(Y)$
(b) $f(X \cup Y) = f(X) \cup f(Y)$,
(c) If $X \subseteq Y$ then $f(X) \subseteq f(Y)$,
(d) $X \subset f^{-1}(f(X))$ and find an example for which we have $f(f^{-1}(X)) \subsetneq X$.

(16) Write down a bijective function between $\mathbb{Z}$ and E, where E are all even numbers of $\mathbb{Z}$.

(17) Let $f : A \to B$, and X be a subset of A and Y be a subset of B. Prove:
(a) If f is surjective then $f(f^{-1}(Y)) = Y$.
(b) If f is injective then $f^{-1}(f(X)) = X$.

(18) Prove that if $f : A \to B$ and $g : B \to C$ are injective/surjective then $g \circ f$ is injective/surjective.

(19) Let A be a set. Define an injective function from A and $P(A)$.

(20) Prove that there cannot be an injective function $f : A \to B$ where $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$

(21) Prove that there cannot be a surjective function $f : A \to B$ where $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$.

(22) Find (with proof)
(a) $\bigcap_{n=1}^{\infty}[1, 1 + \frac{1}{n})$,
(b) $\bigcap_{n=1}^{\infty}(0, \frac{1}{n})$,
(c) $\bigcap_{n=1}^{\infty}[n, n^2]$,
(d) $\bigcup_{n=1}^{\infty}[n, n^2]$.

(23) Write the following sets as intervals or union of intervals
(a) $\{x \in \mathbb{R} | x^1 - 2 \geq 1\}$
(b) $\{x \in \mathbb{R} | \sin x \geq 0\}$
(c) $\{x \in \mathbb{R} || \cos x| < 1/2\}$