

Overview of the Things to know for the Final

General: The final will be comprehensive, nevertheless it will more concentrate on material covered after the midterm. The final will be slightly longer than the midterm, but it will have a similar format. It will ask

- for recitations of theorems and definitions,
- to do some easy problems using these theorems and definitions,
- to do old homework problems.

Material covered after midterm (please go also through the preparation sheet for midterm)

(1) Product spaces

- Definition of product space (Definition 5.22 and Notation 5.23) and product topology (Definition 5.24 and Definition 5.28)
- Projections, continuity and openness of projections (Theorem 5.30, know proof)
- Continuity of $f : Y \rightarrow \prod_{i \in \mathbb{I}} X_i$ (Theorem 5.32, know proof)
- Problems (1) and (2) on HW due 3/27.

(2) Separation Axioms

- T_0, T_1, T_2, T_3, T_4 , normal regular.
- Know examples of spaces, which are T_0 but not T_1 , T_1 but not T_2 , T_2 but not regular, regular but not T_2 , regular but not normal, normal but not regular.
- Important Theorems: 6.9, 6.12, 6.13, 6.23 (know proof).
- Theorem 6.25 and Urysohn's Lemma (Theorem 6.26, know only statements)
- Problems: (3) and (4) on HW due 3/27 and (1), (2), (3) and (4) on HW due 4/3.

(3) Compact Spaces

- Definition 7.1. Also know what an "open cover" and "sub-cover" is.
- Using only definition of compactness show that $\{1/n : n \in \mathbb{N}\}$ is not compact, but that $\{1/n : n \in \mathbb{N}\} \cup \{0\}$ is compact (with respect to usual topology on \mathbb{R}).
- Important Theorems: 7.3, 7.4, 7.5 7.6 (know proof)
- Heine Borel (Know statement)
- Equivalences for compactness in metric spaces: Theorem 7.16 (Know statements)
- Problems: (1), (2), (3) and (4) HW due 4/24 and "Tube lemma" (problem 7, page 95)

- (4) Local compact spaces.
- Definition 8.1. Examples. Show that Sorgenfrey line and Michael line are not locally compact.
 - Alexandrov 1 point compactification for locally compact spaces. Theorem 8.3 (know construction and proof). Examples in 8.6
 - Theorems 8.4, 8.8 and Corollary 8.7 (know proof).
 - Problems (3) and (4) on HW 4/24.
- (5) Connected spaces
- Definition of disconnected and connected.
 - Equivalent conditions for connected/disconnected: Theorems 9.3, 9.10 (know proof)
 - Connected sets in \mathbb{R} (Theorem 9.6).
 - Important Theorems (know proofs) Theorem 9.8 Corollary 9.9
 - Definition of *mutally separated* and Theorem 9.13 (know proof)
 - Problems 1 and 3, page 116.