Overview of the Things to know for the Midterm

(1) Cardinality:
  - Definition of two sets A and B *are equipotent* or *have same cardinality.* Def 1.17.
  - Theorem of Cantor Bernstein. Theorem 1.18 and Problem 1 on Homework 1. Know proof of "easy implication" (asked for in HW 1).
  - Countable sets. Def 1.20. Show directly (without using Cantor Bernstein) that $\mathbb{Z}$ is countable.
  - Criteria of countability: Theorems 1.22, and 1.27 Cor. 1.28 and1.29. Why is $\mathbb{Q}$ countable? Why is $\mathbb{R}$ not countable?

(2) Metric spaces
  - Definition of a *metric*, Def 2.1
  - Examples of metric spaces: 2.3, 2.4, 2.5, 2.7, 2.9
  - *Interior of a set* A, Def 2.10 .
  - *Open sets* in metric spaces, Def 2.12. Show that the interior of a set is open.
  - Properties of open sets , Theorem 2.13. Know proof. Show that arbitrary unions and finite intersections of open sets are open.
  - *Equivalent metrics,* Def. 2.14, Examples 2.1
  - *Cluster point* of a set A, *derivative of* A, *closure of* A, Def 2.20 . Show that $A \subset B \Rightarrow \overline{A} \subset \overline{B}$
  - *Closed sets* in metric spaces, Def 2.21
  - Properties: 2.24
  - Equivalent description of closure, Theorem 2.25.
  - Equivalences to being closed: Lem.2.22, Theorem 2.23, Theorem 2.30. Know proof.
  - Definition of *convergent sequences*, Def 2.29
  - *Cauchy sequences in metric spaces*, Def 2.33
  - *Completeness* of metric spaces 2.37. Explain: Completeness is a property of the metric not the topology.

(3) *Continuous Functions* on metric spaces
  - Definition of *continuity* (at a point; at all points) Defs 3.1 and 3.3.
  - Equivalences to continuity; Theorems 3.2, 3.4 and 3.6
  - *Uniform continuity.* Def 3.8 Find example of uniformly continuous functions on $\mathbb{R}$ which are not linear. Find example of continuous, but not uniformly continuous functions.

  - *Extension Theorem*,Theorem 3.10. Show that Extension Theorem is wrong if one only assumes continuity, and if one does not assume completeness of target space Y.
(4) Topological spaces
  - Definition of *Topology* and *Topological Spaces*, Defs. 4.1., 4.2.
  - Show that the open sets of a metric space is an example of a topology. Show that examples in 4.4 are topologies.
  - *Interior of a set* Def 4.5. Show that this notion coincides with old definition in case of metric spaces.
  - *Neighborhood of a point* $x \in X$ Def 4.7
  - *Closed sets* Def 4.8
  - Properties of closed sets Thm 4.9. Know the proof.
  - *Derived set* and *Closure of a set* Defs.4.10 4.11 Other characterizations of closure: Thm 4.13.
  - Properties of closure Theorem 4.14. Know the proof.
  - *Basis of a topology* Def 4.15. What conditions must a set of subsets of X satisfy to be the basis of some topology, Thm 4.20,
  - *Basis of neighborhood*, Def. 4.22. What is typical neighborhood basis for metric spaces?
  - *First and second countable*, Def 4.25 Show that metric spaces are first countable. Under which condition are metric spaces second countable.
  - *Lindelöff Property* Def 4.32, 4.33
  - Equivalences for metric spaces. Theorem 4.35
(5) Continuity of maps between topological spaces
  - Definition of *continuous* Function, Defs 5.5, 5.6. Give an equivalent definition of "f is continuous at c" using neighborhood bases (and prove it).
  - Equivalences. Theorem 5.7. Know proof.