FINAL, MATH410

Time and Date: Tuesday, May 12, 8–10 am.
General: One third of the final will, roughly speaking, be on Chapter 8 and 9. two thirds on Chatpters 11 and 12(sections 1,2,3)

As usual the test will (roughly 33

- old homework problems
- asking for defintions
- some easy new problems.

Important things you ought to know:

(1) Recall three different norms on $\mathbb{R}^n$, and that they all three satisfy Theorem 8.6.

(2) Comparison between the different norms: see remark 8.7 (page 231)

(3) Cauchy Schwartz (proof!)

(4) Linear Transformations and the fact that they are bounded (Theorem 8.17)

(5) Definition of open sets and closed sets (Definition 8.20)

(6) Open sets are closed under arbitrary unions and finite intersections Closed sets are closed under finite unions and arbitrary intersections. (Theorem 8.24). Give examples which show that open sets/ closed sets are ¡strong¿ not ¡/strong¿ closed under arbitrary intersections/unions.

(7) Relative open/closed sets (Def 8.26)

(8) Connected sets (Def 8.27 and Remark 8.29). Know how to prove that a set in R is connected if and only if it is an interval. Know how to show that the union of two dijoint closed balls is not conntected.

(9) Interior and closure of a set (Def 8.31) Know how to show main properties (Theorems 8.32, 8.37)

(10) Boundary; Definition 8.34 and Theorem 8.36.

(11) Convergence in $\mathbb{R}^n$: Definition 9.1. Know how to prove convergence theorems 9.2 - 9.7. Most of them are like in the one dimensional case. Bolzano - Weierstrass in $\mathbb{R}^n$.

(12) Equivalent description of closed set: Theorem 9.8 (proof).

(13) Open coverings and compact sets (Definitions 9.9). Important Theorem: 9.11 (proof)

(14) Limit of functions: Definition 9.13 and Theorem 9.14 (similar to one dimensional case). Know how to verify limits (Exercises 269/1,2,3).

(15) Continuous Functions. Definition 9.22 and equivalent conditions: Theorems 9.25 and 9.26 (proof).

(16) Compactness and Continuity: Theorems 9.29, 9.32 and 9.33 (proof).

(17) Differentiability:
- Partially differentiable (pages 321)
- Differentiable
- Continuously partially differentiable 11.12
- Higher order partially differentiable Def 11.1

(18) Examples concerning different types of differentiability: Example 11.3, Example 11.18, $f(x, y) = xy/(x^2 + y^2)$, Exercises 2, 5, 6, 7,9.

(19) Differentiablity - Partial Differentiability and how to compute derivatives: Theorem 11.15 (proof)

(20) Interchanging derivatives: Theorem 11.2.

(21) Interchanging limits/ derivatives with integration: 11.4 and 11.5.

(22) Rules of Differentiation (in particular product rule): Theorem 11.20 (proof) Problems: 347/1,2.

(23) Definition of a tangent plane, and Theorem 11.22. Problems 347/5,6,7.

(24) The Chain Rule: Theorem 11.28. Problems 350/ 1,3,5.

(25) The Mean Value Theorem: Theorem 11.31 and Corollaries 11.33 11.34. Problems: 357/1.

(26) $p - th$-total differential of a function $f : V \to R$ (page 355) and Taylor formula (Theorem 11.37) Problems 357/ 3,4.

(27) Derivative of inverses (Inverse Function Theorem): Theorem 11.41 (be sure to be able to state it!). Problems: 367/1,7.

(28) The Implicit Function Theorem: Theorem 11.47. Problems 367/2,3.

(29) Definition of local max/min /saddle-points for functions $f : V \to \mathbb{R}$, $V \subset \mathbb{R}^n$ open: Definition 11.50 and 11.53. Assuming f differentiable, what is a necessary condition for loc. extremes.

(30) Second Derivative Test; Theorem 11.56. Problems 378/1

(31) Special case of second derivative test in $\mathbb{R}^2$: Theorem 11.59. Problems 378/2

(32) Local Extreme Points subject to constraints: Definition 11.61, Lagrange Multiplier Method :Theorem 11.63. Problems 378/3

(33) What is a rectangle in $\mathbb{R}^n$ and a grid on a rectangle, what does it mean that one gird is finer than an other , what is $V(E, \mathcal{G})$ and $v(E, \mathcal{G})$? (pages 381,382,383, 390). Problems 392/1

(34) Jordan region and volume of Jordan region: Definitions 12.3 and 12.4. Know how to show that the definition of the volume does not depend on the rectangle one embeds the set E.

(35) Equivalent condition for Jordan region of volume 0; Theorem 12.8, Corollaries 12.9 and 12.10.

(36) Inner volume and outer volume : Definition 12.13 and Theorem 12.14.Problems 393/2,3,4,5.

(37) Definition of upper/lower sums and upper/lower integrals of functions $f : E \to \mathbb{R}$ bounded, $E \subset \mathbb{R}^n$ Jordan region: Defintion 12.15. Remark 12.16

(38) Riemann integrability and equivalent condition: Definition 12.17 and Remark 12.18.

(39) Riemann integrability of continuous functions (know proof): Theorem 12.21.

(40) Properties of Riemann integral: Theorem 12.22, 12.23, 12.24, 12.25. Problems 405/1,2,3,4.