Theorems and Definitions you need to be able to state (correctly) for Exam I:

**Definition of a Riemann sum for Functions**

Assume $f : [a, b] \to \mathbb{R}$ is a function on an interval $[a, b]$, assume that $P = (x_0, x_1, \ldots, x_n)$ is a partition of $[a, b]$, which means that $a = x_0 < x_1 < x_2 < \ldots x_n$, and assume that $\left(x^*_1, x^*_2, \ldots, x^*_n\right)$ are intermediate points, which means that $x^*_i \in [x_{i-1}, x_i]$, for $i = 1, 2, \ldots n$.

Then

$$R(P, (x^*_i), f) = \sum_{i=1}^{n} f(x^*_i)(x_i - x_{i-1}),$$

is defined to be the Riemann sum for $f$, the partition $P$ and the intermediate points $(x^*_i)$.

**The Mesh of a Partition**

For a partition $P = (x_0, x_1, \ldots, x_n)$ of $[a, b]$ we call

$$||P|| = \max_{i=1,2,\ldots,n} |x_i - x_{i-1}|$$

the mesh of $P$.

**Definition of the Definite Integrals for Continuous Functions**

Assume $f : [a, b] \to \mathbb{R}$ is a function on an interval $[a, b]$, Then

$$\lim_{\|P\| \to 0} \sum_{i=1}^{n} f(x^*_i)(x_i - x_{i-1}),$$

exists and it is called the definite integral of $f$ from $a$ to $b$ and is denoted by

$$\int_{a}^{b} f(x)dx.$$

**Fundamental Theorem of Calculus, Part I**

Suppose that $a$ and $b$ are real numbers with $a < b$. Assume that $f$ is continuous on $[a, b]$, and define

$$F(x) := \int_{a}^{x} f(t) dt, \quad a \leq x \leq b.$$ 

Then $F'(x) = f(x)$ for every $a < x < b$, $F'_+(a) = f(a)$, and $F'_-(b) = f(b)$. 
Fundamental Theorem of Calculus, Part II

Suppose that \( a \) and \( b \) are real numbers with \( a < b \). Assume that \( f \) is continuous on \([a, b]\). Let \( G \) be a function satisfying the following conditions: \( G \) is continuous on \([a, b]\), and \( G'(x) = f(x) \) for every \( a < x < b \). Then

\[
\int_a^b f(x) \, dx = G(b) - G(a).
\]

Leibniz’s Formula

Suppose that \( f \) is continuous, and that \( u \) and \( v \) are differentiable. Then

\[
\frac{d}{dx} \left[ \int_{u(x)}^{v(x)} f(t) \, dt \right] = f(v(x))v'(x) - f(u(x))u'(x).
\]

Mean Value Theorem for Integrals

Suppose that \( a \) and \( b \) are real numbers with \( a < b \). Assume that \( f \) is continuous on \([a, b]\). There exists a point \( c \) in the interval \([a, b]\) such that

\[
\frac{1}{b - a} \int_a^b f(t) \, dt = f(c).
\]

Theorem on Integration by parts

Suppose that \( a \) and \( b \) are real numbers with \( a < b \). Let \( f \) and \( g \) be differentiable on \([a, b]\), and assume that \( f' \) and \( g' \) are continuous on \([a, b]\). Then

\[
\int_a^b f'(x)g(x) \, dx + \int_a^b f(x)g'(x) \, dx = f(b)g(b) - f(a)g(a).
\]