Math 251.512
Exam 1, version A
9/19/08

1. For the function \( f(x, y) = \sqrt{16 - 4x^2} + \sqrt{9 - y^2}, \)
   (a) \( 6 \text{ pts.} \) sketch the domain.
   (b) \( 6 \text{ pts.} \) find the range.

2. \( 8 \text{ pts.} \) Find two unit vectors perpendicular to both \( \langle 2, 1, -1 \rangle \) and \( \langle -1, 2, 1 \rangle \).

3. \( 10 \text{ pts.} \) Find the equation of the plane which contains the line \( x = 1 + 2t, \)
   \( y = 2 - t, z = 3t \) and is perpendicular to the plane \( 3x + y + 2z = 5. \)

4. \( 10 \text{ pts.} \) At what point(s) (if any) on the curve \( x = t, \ y = t^2, \ z = t^3 \) is
   the tangent vector perpendicular to \( \mathbf{i} + \mathbf{j} - \mathbf{k} \)?

5. \( 8 \text{ pts.} \) What is the area of the triangle with vertices \( (1, 0, 3), \ (0, 1, 2), \)
   and \( (4, 1, 2) \)?

6. A particle starts at the point \( (0, 0, 0) \) with initial velocity \( \mathbf{v}(0) = \mathbf{j} + 2\mathbf{k} \)
   and moves with acceleration \( \mathbf{a}(t) = -\mathbf{i} + \mathbf{j} \).
   (a) \( 6 \text{ pts.} \) What is its position as a function of \( t? \)
   (b) \( 6 \text{ pts.} \) Where does it cross the plane \( x = -1? \) (Consider only \( t \geq 0. \))

7. \( 10 \text{ pts.} \) Find the equation of a sphere, given that one of its diameters
   has endpoints \( (1, 1, 3) \) and \( (3, -3, 7) \).

8. \( 8 \text{ pts.} \) Sketch the polar curve \( r = \cos 2\theta, \ 0 \leq \theta \leq \pi. \) (Note the restriction
   on \( \theta! \))

9. \( 10 \text{ pts.} \) What is the length of the curve \( x = 2t, \ y = 3\sin t + \cos t, \)
   \( z = \sin t - 3\cos t \) from \( (0, 1, -3) \) to \( (\pi, 3, 1) \)? (If you’re trying to do a
   difficult integral, you’ve made a mistake.)
10. (12 pts.) For each of the following equations, determine which surface on the following page is its graph. In all the graphs, the axes are oriented as shown:

(a) \( x^2 + y^2 = z^2 \)
(b) \( x^2 - y^2 + z^2 = 1 \)
(c) \( z = x^2 - y^2 \)
(d) \( z = x^2 + y^2 \)
(e) \( x^2 = y^2 + z^2 \)
(f) \( x^2 + y^2 - z^2 = 1 \)