Linear Algebra for Differential Equations
Math 309.501, 309.502
Spring, 2014

Section 501: MWF 10:20-11:10 (BLOC 164)
Section 502: MWF 11:30-12:20 (BLOC 164)

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Office: 629C Blocker
e-mail: tvogel@math.tamu.edu
eCampus: There will be many useful things posted on this page including lecture notes, assignments, and all material handed out in class.
Office hours: W-F, 2:00-4:00 or by appointment (talk to me after class). Just dropping in is okay, too, although I can’t guarantee that I’ll be there outside of my office hours. Also, I’m an adviser for the Math department, so during my advising hours (M,T 2-4), students who want advising have priority. If you’re running into problems, the sooner you come by and get them fixed, the happier we’ll both be. I often keep my office door closed, because the hallway outside can get noisy. That doesn’t mean I’m not glad to see students: just knock.

Texts: Linear Algebra with Applications, 8th edition, by Leon, and my own lecture notes on Fourier series and Bessel functions, which I will make available as a pdf in eLearning (this is also a required text). I’m also making my lecture notes on linear algebra available in “fill-in-the-blank” form as a pdf which you can download and have printed at any copy shop. This is a suggested text, not required.

Other linear algebra courses: Be aware that you can’t get credit in more than one of Math 304, Math 309, Math 311, and Math 323. That’s because there is quite a bit of overlap of material.

Prerequisites: Third semester calculus (Math 221, 251, or 253) and Math 308 (ordinary differential equations), or concurrent enrollment in 308. If you don’t satisfy these prerequisites, you should not be in this course.

Course content: Systems of linear equations, matrices and determinants. Vector spaces and linear transformations, and applications of these, including eigenvalues and eigenvectors, diagonalization, inner product spaces. Separation of variables in solving partial differential equations. Fourier series and Bessel functions.

Course objectives: If a student successfully completes this course, I expect that they can do the following.

- Leon, Chap. 1
  - Find general solutions to systems of linear equations using Gaussian and Gauss-Jordan elimination.
  - Perform basic matrix arithmetic and matrix algebra, including inverting matrices. Be able to prove basic propositions concerning inverses, and transposes of matrices.

- Chap. 2
- Compute determinants of square matrices by cofactors. Explain and be able to use the connection between determinant and invertibility of a matrix.

- Chap. 3
- Explain the definition of a vector space.
- Verify whether a given set, with given operations, is a vector space.
- Given a subset of a vector space, be able to determine if it is a subspace.
- Explain the concepts of linear combination and spanning sets of vectors, and be able to prove basic propositions concerning these concepts.
- Explain the concept of linear independence of a set of vectors. Given a set of vectors in a vector space, be able to determine whether this set is a linearly independent set. Be able to prove basic propositions concerning linear dependence and independence.
- Explain the concept of a basis for a vector space. Find bases of given vector spaces.
- Explain the concept of a coordinate vector with respect to a given basis. Find the transition matrix from one basis to another.
- Explain what a row space and a column space of a given matrix is, the relationship between the consistency of a linear system and the column space of the coefficient matrix. Be able to state the rank-nullity system and use it to prove basic propositions.

- Chap. 4
- Explain the concept of a linear transformation, and be able to determine whether is given transformation is linear. Be able to prove basic propositions concerning linear transformations.
- Be able to find the matrix representation of a linear transformation with respect to given bases.
- Explain the concept of similar matrices, and how this relates to matrix representation of linear transformations.

- Chap. 6
- Explain the concept of eigenvalue and eigenvector of a matrix and of a linear operator. Be able to find eigenvalues and eigenvectors of given matrices and linear operators. Be able to prove basic propositions concerning eigenvalues and eigenvectors.
- Apply theory of eigenvalues and eigenvectors to solve systems of linear ordinary differential equations.
- Be able to diagonalize a matrix.
• Chap. 5

  – Find scalar product of two vectors in \( \mathbb{R}^n \), find angle between two vectors in \( \mathbb{R}^n \).
  – Explain the concept of an inner product on a vector space.
  – Explain the concept of an orthonormal set of vectors, and be able to apply basic results.

• Fourier series and Bessel function notes

  – Chap. 1. be able to classify partial differential equations as linear, nonlinear, elliptic, hyperbolic, parabolic. Be able to solve certain PDE’s by integration.
  – Chap. 2. Be able to solve partial differential equations using separation of variables.
  – Chap. 3. Compute Fourier series corresponding to given functions. Compute half-range Fourier series and double Fourier series.
  – Chap. 4. Apply Fourier series to produce series solutions to boundary value problems.
  – Chap. 5: Explain the definition of Bessel functions of the first and second kind. Be able to verify basic identities for Bessel functions. Be able to apply Bessel-Fourier series to solve boundary value problems.

**Grading:**

<table>
<thead>
<tr>
<th>Component</th>
<th>Points</th>
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<tbody>
<tr>
<td>3 tests, 100 points each</td>
<td>300 pts.</td>
</tr>
<tr>
<td>Final exam</td>
<td>180 pts.</td>
</tr>
<tr>
<td>Homework</td>
<td>100 pts.</td>
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<tr>
<td>Attendance</td>
<td>20 pts.</td>
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**Final grade:**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Points</th>
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<tbody>
<tr>
<td>A</td>
<td>540-600</td>
</tr>
<tr>
<td>B</td>
<td>480-529</td>
</tr>
<tr>
<td>C</td>
<td>420-479</td>
</tr>
<tr>
<td>D</td>
<td>360-419</td>
</tr>
<tr>
<td>F</td>
<td>0-339</td>
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</tbody>
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(I have been known to curve final grades if I feel that it is warranted.)

**Homework:** Homework will be typically be due on Fridays at 4 pm, except for weeks during which there is a test. (The last homework will be due on a Wednesday to give the grader time to get it back.) Late homework that is a week or less late will have 20% deducted. Homework that is more than a week late will not be accepted. Your lowest homework score will be dropped and a percentage computed from the rest in determining the homework portion of your final grade (i.e., homework points may not equal test points). Discussing homework with me or your classmates is okay, but passing off someone else’s work as your own is plagiarism, and is a violation of the Aggie Honor Code. Similarly, copying answers from a solutions manual is plagiarism. If a zero is
given on homework as a penalty for cheating, that homework will not be dropped in determining the homework grade.

**Where to go for help:** I have been told that there may be a help session room for this class, depending on availability of graduate students. If this is, I will announce time and place as soon as it’s decided. I’m always glad to help students during my office hours (*That’s what they’re there for!*), and other times by appointment. Also, the math department keeps a list of tutors (ask at the department office, in BLOC 227).

**Tests:** Tests will be given on 2/14 (Friday), 3/21 (Friday), and 4/16 (Wednesday) with reviews to be given on 2/11 (Tuesday), 3/18 (Tuesday), and 4/13 (Sunday) with locations and times to be announced. The sections covered on a test will be announced well before the test is given. Complaints about test or homework grades must be made within two weeks of when the material is returned to the class. Beyond that time, the grade will not be changed. If you must miss a test due to a University excused absence (see http://student-rules.tamu.edu/rule07) notify me before the test if possible, but at the latest by the end of the second working day following the absence. Otherwise I’m under no obligation to write a make-up exam. **Lowest grade:** I will give you a break on your lowest test grade if you show me that you understand the corresponding material on the final. Specifically, I will find your lowest test grade and compute the percentage of points obtained on questions on the final related to your lowest test. If that percentage is higher than your grade on the hour exam, I’ll raise the grade on your lowest test, but no higher than your second lowest test grade. This is designed specifically to help the student who does well on two of the hour exams, but not so well on the third. If you do badly on two exams, you’re on your own. If a zero is given on a test as a penalty for cheating, then the “lowest grade” policy outline above will not be applied: that zero will not be changed.

**Bluebooks:** Please turn in to me four 8 1/2 by 11 blank bluebooks (not even your name on them) before the day of the first exam.

**Attendance:** You have two free unexcused absences. After that, you lose 1 point for each unexcused absence, for a maximum of 20 points lost. These are the easiest points in the class, so if you cut class, you get no sympathy from me. I don’t mind if you attend the other lecture that day, as long as there are seats available, and as long as the classes are covering the same material. Take exams with your section, however.

**Final exams are**

<table>
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<tr>
<th>Section (MWF)</th>
<th>Time</th>
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<tr>
<td>501</td>
<td>Tuesday, 5/6, 8:00 am–10:00 am</td>
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<tr>
<td>502</td>
<td>Wednesday, 5/7, 10:30 am–12:30 pm</td>
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If you have three finals on one day, I’m willing to let you take the final with the other section; contact me well ahead of time. Otherwise, take it with your own section.

The following schedule is tentative, and may have to be adjusted slightly during the semester.
• 1/13: Systems of linear equations (Leon, 1.1)
• 1/15: Row echelon form. Solution of systems by Gaussian elimination (1.1, 1.2)
• 1/17: Gauss-Jordan elimination. Applications of systems (1.2)
• 1/20: MLK day (reading day)
• 1/22: Matrix arithmetic (1.3)
• 1/24: Matrix arithmetic and matrix algebra (1.3, 1.4)
• 1/27: Inverting matrices and computing determinants (1.5, 2.1)
• 1/29: Properties and applications of determinants (2.2, 2.3)
• 1/31: Introduction to vector spaces (3.1)
• 2/3: Subspaces (3.2)
• 2/5: Spanning sets, linear independence (3.2, 3.3)
• 2/7: Vector spaces of functions and the Wronskian (3.3)
• 2/10: Basis and dimension (3.4)
• Tuesday, 2/11: Review for exam 1, location and time TBA,
• 2/12: Bases and coordinates (3.4, 3.5)
• 2/14: Exam 1.
• 2/17: Change of basis (3.5)
• 2/19: Row space and column space (3.6)
• 2/21: Row space and column space. Definition of linear transformation (3.6, 4.1)
• 2/24: Image, kernel, and range of linear transformations. Matrix representations of linear transformations (4.1, 4.2)
• 2/26: The matrix representation theorem (4.2)
• 2/28: Similar matrices (4.3)
• 3/3: Eigenvalues and eigenvectors (6.1)
• 3/5: Applications to systems of ordinary differential equations. Diagonalization of matrices (6.2, 6.3)
• 3/7: Diagonalization of matrices (6.3)
• 3/10-3/14: Spring Break
• 3/17: Scalar product in \( \mathbb{R}^n \). (5.1)

• Tuesday 3/18: Review for exam 2, location and time TBA.

• 3/19: Inner product spaces. (5.4)

• 3/21: Exam 2.

• 3/24: Normed linear spaces and orthonormal sets (5.4, 5.5)

• 3/26: Applications of orthonormal sets (5.5)

• 3/28: Introduction to boundary value problems. Superposition of solutions (Notes, chapter 1)

• 3/31: Classification of PDE’s. Solution by integration. (chapter 1)

• 4/2: Solution by separation of variables (chapter 2)

• 4/4: Fourier series and the Dirichlet conditions (chapter 3)

• 4/7: Half-range sine and cosine series and double Fourier series (chapter 3)

• 4/9: Series solutions to PDE’s using Fourier series (chapter 4)

• 4/11: Series solutions to PDE’s using Fourier series, continued (chapter 4)

• Sunday, 4/13: Review for exam 3. Location and time TBA.

• 4/14: Introduction to Bessel functions (chapter 5)

• 4/16: Exam 3.

• 4/18: Good Friday (reading day).

• 4/21: Orthogonality and series of Bessel functions (chapter 5)

• 4/23: Application of Bessel functions to boundary value problems (chapter 5)

• 4/25: Application of Bessel functions (continued) (chapter 5)

• 4/28: Application of Bessel functions, start review (chapter 5)

• 4/29: Review (Redefined day. Attend Friday classes.)

• Tuesday 5/6, 8:00 am-10:00 am: Final for section 501 (MWF 10:20-11:10)

• Tuesday 5/7, 10:30 am – 12:30 pm: Final for section 502 (MWF 11:30-12:20)