Exam 1, version A
Math 171.501
9/29/09

An unsupported answer is a wrong answer!

Do not use l'Hôpital's rule on this test. (If you don't know what that is, don't worry.)

1. (10 pts.)
   (a) Suppose that f is defined in an open interval containing a, except possibly at a itself. State the formal δ, ε definition of $\lim_{x\to a} f(x) = L$.
   (b) Use the formal definition to prove that $\lim_{x\to 2} 2x^2 - 8x = -8$.

2. (15 pts.) Evaluate these limits, showing your work:
   (a) $\displaystyle\lim_{x\to\infty} \frac{x+1}{\sqrt{3x^2+2x}}$.
   (b) $\displaystyle\lim_{x\to 2} \frac{\sqrt{x+7}-3}{x-2}$.
   (c) $\displaystyle\lim_{x\to\frac{1}{2}^-} \frac{|2x-1|}{2x^2+x-1}$.

3. (5 pts.) For what number x is does the component of $\langle -1, x\rangle$ along the vector $\langle 3, 4\rangle$ equal 2?

4. (10 pts.) What is the domain of $g(x) = \sqrt{9-4x^2}$? What is the range of g? Explain your answers.

5. (10 pts.) If $\vec{r}(t) = \langle t^2+3t, 2t\rangle$, determine $\displaystyle\lim_{h\to 0} \frac{1}{h}\left(\vec{r}(1+h) - \vec{r}(1)\right)$.

6. (10 pts.) Solve the inequality $\dfrac{x}{x+1} > 2$. (Your answer will be an interval or union of intervals.)

7. (10 pts.) An object is moving in the xy-plane, and its position after t seconds is $\vec{r}(t) = (t+3)\,\vec{i} + (6t - t^2)\,\vec{j}$.
   (a) Does the object go through the point $(5, 2)$? Why or why not?
   (b) Find an equation in x and y whose graph is the path of the object.

8. (10 pts.) Suppose that $\sin\theta = \dfrac{2}{3}$, and that $\frac{\pi}{2} < \theta < \pi$. Find $\cos\theta$ and $\tan\theta$.

9. (10 pts.) A function f is defined by

$$f(x) = \begin{cases} 2x-3, & x < 2 \\ a, & x = 2 \\ bx+4, & x > 2. \end{cases}$$

   What values of a and b will make f continuous at 2? Explain your answer.

10. (10 pts.) In the figure, let $\overrightarrow{a}$ be the vector from O to A, and let $\overrightarrow{b}$ be the vector from O to B. Suppose that C and D divide the line segment $\overline{AB}$ into equal thirds. Find the vector which starts at 0 and goes to D in terms of $\overrightarrow{a}$ and $\overrightarrow{b}$.

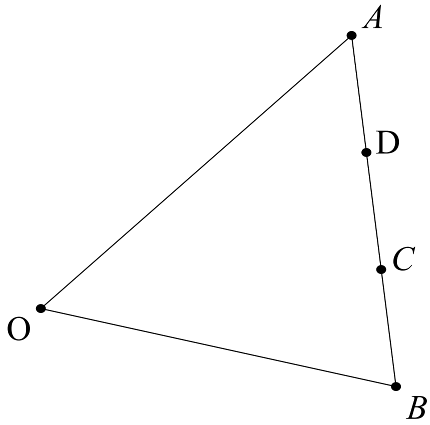

Figure for problem 10