1.

(a) For every $\varepsilon > 0$ there exists a $\delta > 0$ such that $0 < |x - a| < \delta$ implies $|f(x) - L| < \varepsilon$.

(b) I'll give only the proof; the scratchwork to find δ in terms of ε is pretty much the proof written backwards. Let $\varepsilon > 0$ be an arbitrary positive number. Set δ to be 2ε. Then $0 < |x - 4| < 2\varepsilon$ implies $|x - 4| < 2\varepsilon$. From this, it follows that $\frac{1}{2}|x - 4| < \varepsilon$, hence $\left|\frac{-1}{2}\right| |x - 4| < \varepsilon$. Multiplying through, $\left|2 - \frac{x}{2}\right| < \varepsilon$, therefore $\left|\left(5 - \frac{x}{2}\right) - 3\right| < \varepsilon$ as required. Note: many people were sloppy with their absolute values and got -2ε for δ. That can't be right: δ must be positive (see the definition) and -2ε is negative.

2.

(a) Since x is approaching 2 from the left, $x < 2$, so $x - 2 < 0$, and $|x - 2| = -(x - 2)$ for those x's. Therefore,

$$\begin{aligned} \lim_{x \to 2^-} \frac{x^2 + x - 6}{|x - 2|} &= \lim_{x \to 2^-} \frac{x^2 + x - 6}{-(x - 2)} \\ &= \lim_{x \to 2^-} \frac{(x - 2)(x + 3)}{-(x - 2)} \\ &= \lim_{x \to 2^-} \frac{(x + 3)}{-1} = -5. \end{aligned}$$

(b) Multiply through by the conjugate of the denominator:

$$\begin{aligned} \lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} &= \lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} \left(\frac{\sqrt{x} + 2}{\sqrt{x} + 2}\right) \\ &= \lim_{x \to 4} \frac{(x - 4)(\sqrt{x} + 2)}{(x - 4)} \\ &= \lim_{x \to 4} \sqrt{x} + 2 = 4. \end{aligned}$$

(c) Inside the square root, multiply top and bottom by $\frac{1}{x^2}$:

$$\begin{aligned} \lim_{x \to \infty} \sqrt{\frac{x^2 + 3x + 7}{9x^2 + x + 2}} &= \lim_{x \to \infty} \sqrt{\frac{(x^2 + 3x + 7)\frac{1}{x^2}}{(9x^2 + x + 2)\frac{1}{x^2}}} \\ &= \lim_{x \to \infty} \sqrt{\frac{1 + \frac{3}{x} + \frac{7}{x^2}}{9 + \frac{1}{x} + \frac{2}{x^2}}} \\ &= \sqrt{\frac{1}{9}} = \frac{1}{3}. \end{aligned}$$

3. (a) We have

$$\begin{aligned} \lim_{x \to 1} f(x) &= \lim_{x \to 1} \frac{x^2 - 1}{x^2 + 5x - 6} \\ &= \lim_{x \to 1} \frac{(x + 1)(x - 1)}{(x + 6)(x - 1)} \\ &= \lim_{x \to 1} \frac{x + 1}{x + 6} = \frac{2}{7}. \end{aligned}$$

Thus the limit exists and equals $\frac{2}{7}$.

(b) $f(x)$ is not continuous at 1, since $f(1)$ is defined to be $\frac{1}{2}$, therefore $\lim_{x \to 1} f(x) \neq f(1)$.

4. (a) First, $2\vec{a} + \vec{b} = 2\langle 2, 4\rangle + \langle -1, 1\rangle = \langle 3, 9\rangle$, so $\left|2\vec{a} + \vec{b}\right| = |\langle 3, 9\rangle| = \sqrt{9 + 81} = \sqrt{90}$.

(b) From the formula for angle between vectors,

$$\begin{aligned}\cos\theta &= \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\left|\vec{b}\right|}\\ &= \frac{2(-1)+4(1)}{\sqrt{4+16}\sqrt{1+1}}\\ &= \frac{2}{\sqrt{40}},\end{aligned}$$

which simplifies to $\frac{\sqrt{10}}{10}$, although I didn't require any simplification.

(c) We have

$$\begin{aligned}\text{proj}_{\vec{a}}\vec{b} &= \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a}\\ &= \frac{\langle 2,4\rangle\cdot\langle -1,1\rangle}{\langle 2,4\rangle\cdot\langle 2,4\rangle}\langle 2,4\rangle\\ &= \frac{2}{20}\langle 2,4\rangle\\ &= \left\langle \frac{1}{5},\frac{2}{5}\right\rangle.\end{aligned}$$

5. Using the formula for $\sin 2\theta$, the equation becomes $2\sin\theta\cos\theta = \sqrt{2}\cos\theta$. Putting everything on the left and factoring, it becomes $\cos\theta\left(2\sin\theta-\sqrt{2}\right)=0$. This product is zero precisely when a factor is zero, so we need all θ's for which either $\cos\theta = 0$ or $2\sin\theta-\sqrt{2}=0$. These are $\theta=\frac{\pi}{4}, \frac{\pi}{2}, \frac{3}{4}\pi, \frac{3}{2}\pi$.

6.

(a) $x = 3+2t$, $y = 1+t$.

(b) First, I find two points on the line by plugging in different t's: $(3,1)$ is on the line (that's $t=0$), and also $(5,2)$ (that's $t=1$). The slope of the line is $\frac{2-1}{5-3}=\frac{1}{2}$, and so by the point-slope form of the equation of a line, the line is $y-1=\frac{1}{2}(x-3)$, which simplifies to $y=\frac{1}{2}x-\frac{1}{2}$.

7. Consider the parameterized curve $x=\frac{1}{1-t}$, $y=\frac{1}{1+t}$, $-\infty<t<\infty$.

(a) No: the only t value which makes y equal to 1 is $t=0$, but that doesn't make x equal 2.

(b) Solve the first equation for t: $\frac{1}{x}=1-t$, so $t=1-\frac{1}{x}$. Plug this into the equation for y:

$$\begin{aligned}y &= \frac{1}{1+\left(1-\frac{1}{x}\right)}=\frac{1}{2-\frac{1}{x}}\\ &= \frac{x}{2x-1},\ x\neq 0.\end{aligned}$$

8. The graph is:

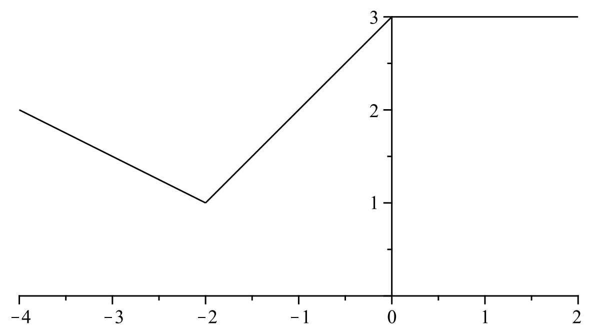

Figure for problem 8

9. The vector from O to C is $\frac{1}{2}\overrightarrow{a}$, so by the picture for vector subtraction, the vector from C to B is $\overrightarrow{b} - \frac{1}{2}\overrightarrow{a}$.