Exam 1, version A
Math 171.501
2/19/09

An unsupported answer is a wrong answer!

1. (a) (5 pts.) Suppose that f is defined in an open interval containing a, except possibly at a itself. State the formal δ, ε definition of $\lim_{x\to a} f(x) = L$.

   (b) (5 pts.) Use the formal definition to prove that $\lim_{x\to 4}\left(5 - \frac{x}{2}\right) = 3$.

2. Evaluate the following limits:

   (a) (5 pts.) $\lim_{x\to 2^-} \dfrac{x^2+x-6}{|x-2|}$.

   (b) (5 pts.) $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$.

   (c) (5 pts.) $\lim_{x\to\infty} \sqrt{\dfrac{x^2+3x+7}{9x^2+x+2}}$.

3. Define $f(x)$ by

$$f(x) = \begin{cases} \dfrac{x^2-1}{x^2+5x-6}, & x \neq 1 \\ \\ 1/2, & x = 1 \end{cases}.$$

   (a) (5 pts.) Does $\lim_{x\to 1} f(x)$ exist? If so, what is it?

   (b) (5 pts.) Is $f(x)$ continuous at 1? Why or why not?

4. If $\vec{a} = \langle 2, 4\rangle$ and $\vec{b} = \langle -1, 1\rangle$, find:

   (a) (5 pts.) $\left|2\vec{a} + \vec{b}\right|$.

   (b) (5 pts.) $\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.

   (c) (5 pts.) The vector projection of $\vec{b}$ onto $\vec{a}$.

5. (10 pts.) Find all angles θ in the interval $[0, 2\pi]$ which satisfy $\sin 2\theta = \sqrt{2}\cos\theta$.

6. A line has vector equation $\vec{r}(t) = \langle 3, 1\rangle + t\langle 2, 1\rangle$.

   (a) (5 pts.) Write parametric equations for this line.

   (b) (5 pts.) Write the equation of the line in slope-intercept form (i.e., $y = mx + b$ for the correct m and b).

7. Consider the parameterized curve $x = \frac{1}{1-t}$, $y = \frac{1}{1+t}$, $-\infty < t < \infty$.

   (a) (5 pts.) Does this curve go through the point $(2, 1)$? Why or why not?

   (b) (5 pts.) Find the Cartesian equation of the curve (i.e., the equation in x and y).

8. (10 pts.) The function $f(x)$ is defined on $[-1, 2]$, and its graph is given below. Sketch the graph of $y = 1 + f\left(\frac{x}{2} + 1\right)$. (Label integers along the x and y axes).

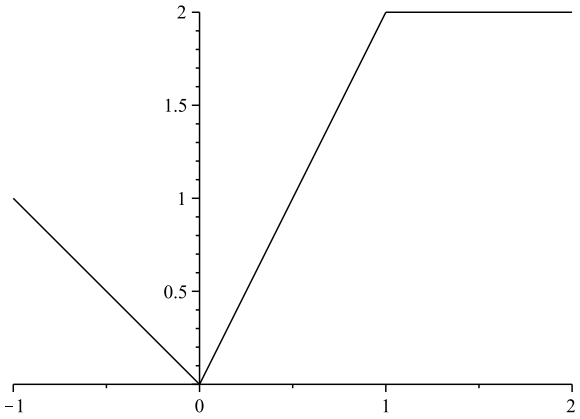

Figure for problem 8

9. (10 pts.) In the figure, let $\overrightarrow{a}$ be the vector from O to A, and let $\overrightarrow{b}$ be the vector from O to B. Let C be the midpoint of the side of the triangle from O to A (misprint corrected). Find the vector which starts at C and goes to B in terms of $\overrightarrow{a}$ and $\overrightarrow{b}$.

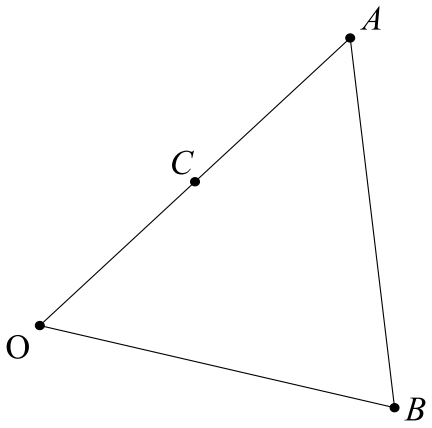

Figure for problem 9