

### Dimensional analysis problems

Math 647

due 2/6

1. Why do stringed musical instruments have strengths of different lengths and thicknesses? Assume that the fundamental frequency  $\omega$  of vibration of a string depends on its length  $l$ , mass per unit length  $\mu$ , and tension (force)  $F$  on the string. Prove that  $\omega$  must be proportional to  $\frac{\sqrt{F/\mu}}{l}$ .
2. We now want to include frictional effects and the initial angle in analyzing pendulums. As in the lecture notes, the length of the pendulum is  $l$  and its mass is  $m$ .
  - (a) Suppose that the frictional force is due primarily to air and is proportional to  $v^2$  with constant of proportionality  $k$ . Let  $\tau$  be the time required for the pendulum to reach half its initial amplitude  $\theta$ . Determine the dimensions of  $k$ . Show that

$$\tau = \sqrt{\frac{l}{g}} G\left(\theta, \frac{kl}{m}\right)$$

for some function  $G$ .

- (b) Deduce a similar result if the frictional force is assumed to be proportional to  $v$ .
3. How long should you roast a turkey? Typically, cookbooks give instructions such as: "set the oven to  $T_0$  degrees and allow  $n$  minutes per pound for cooking." This is saying that the cooking time is proportional to the mass of the turkey. Does this make sense from the point of view of dimensional analysis? A piece of meat is cooked when its minimum internal temperature reaches a certain value dependent on the type of meat and the desired doneness. The variables in the problem are: the cooking time  $t$ , the difference in temperature  $\Delta T_r$  between the raw meat and the oven, the difference in temperature  $\Delta T_c$  between the cooked meat and the oven, some characteristic dimension  $l$  of the meat, and the coefficient  $\kappa$  of heat conductance for the turkey. The dimensions of  $\kappa$  are  $L^2T^{-1}$ .

- (a) Show that

$$t = \frac{l^2}{\kappa} \phi\left(\frac{\Delta T_r}{\Delta T_c}\right).$$

In particular, show that for geometrically similar turkeys differing only in size (i.e., the initial and desired temperatures are the same, along with density and coefficient of heat conductance), that the cooking time should be proportional to mass raised to the two-thirds power.

- (b) Suppose we have a mutant 100 pound turkey, and a very, very big oven. How long should I cook it at  $325^\circ$ ? Use the table at the National Turkey Federation's website: <http://www.eatturkey.com/consumer/cookinfo/turroast.html> to estimate the constant of proportionality, explaining your reasoning. Assume that the turkey is not stuffed, and that we start it at refrigerator temperature, as they suggest. For fun, you might also want to look at [http://www.theonion.com/content/news/thousands\\_gather\\_for\\_stuffing\\_of](http://www.theonion.com/content/news/thousands_gather_for_stuffing_of), although since this turkey is stuffed, our results don't apply.