Homework 1

Problem 1. Using the following definition of k^* show that k^* is a symmetric and positive definite tensor.

$$k^* \cdot e = \langle k \nabla \phi_e \rangle,$$

where $div(k\nabla\phi_e) = 0$ and $\phi_e = e \cdot x + per$. Does this hold for $\phi_e = e \cdot x$?

Problem 2. Assume

$$k^* \cdot e = \langle k \nabla \phi_e \rangle,$$

where $div(k\nabla\phi_e) = 0$ and $\phi_e = e \cdot x$. Show that k^* is independent of the choice e, by expressing k^* for any $e = (e_1, e_2)$ through $e = (1, 0)$ and $e = (0, 1)$.

Problem 3. Compute k^* for

$$k(x_1, x_2) = exp(2 * sin(4 * (x_1 - x_2)))$$

Compute it using $e \cdot x$ and mixed boundary conditions and compare the results and state whether k^* depends on local boundary conditions.