| | | | | | |
|---|---|---|---|---|---|
| 1 | / 8 | 5 | /16 | 8 | /10 |
| 2 | / 8 | 6 | /12 | 9 | /12 |
| 3 | / 8 | 7 | / 8 | 10 | /13 |
| 4 | /10 | Total | | | /105 |

Name________________________________

MATH 172 Honors    Exam 3    Spring 2024

Section 200    P. Yasskin

Points indicated. Part credit possible. Show all work.

**1**. (8 points) Compute the following limits.

**a**. (2 pts) $\lim\limits_{n\to\infty} \dfrac{n^2 - 3n + 2}{n + 1}$

**b**. (2 pts) $\lim\limits_{n\to\infty} \sqrt[n]{3n + 1}$

**c**. (4 pts) $\lim\limits_{n\to\infty} \left(2 + \dfrac{3}{n}\right)^{4n}$

**2**. (8 points) Find a power series for $\arctan(x) = \displaystyle\int_0^x \frac{1}{1 + t^2}\, dt$ centered at $x = 0$.

Hint: Write a series for each of the following:

**a**. $\dfrac{1}{1 - r} = \displaystyle\sum_{n=0}^{\infty}$ ____________________

**b**. $\dfrac{1}{1 + t^2} = \displaystyle\sum_{n=0}^{\infty}$ ____________________

**c**. $\displaystyle\int \frac{1}{1 + t^2}\, dt = \sum_{n=0}^{\infty}$ ____________________ $+C$

**d**. $\displaystyle\int_0^x \frac{1}{1 + t^2}\, dt = \sum_{n=0}^{\infty}$ ____________________

**3**. (8 points) Compute the sum of each series. Simplify each to a single rational number.

**a**. (4 pts) $S = \sum_{n=1}^{\infty} \frac{3^n + 4^n}{12^n}$ HINT: First split the terms into the sum of two fractions.

**b**. (4 pts) $S = \sum_{n=2}^{\infty} \frac{1}{n(n+1)}$ HINT: First find the partial fraction expansion for $\frac{1}{n(n+1)}$.

**4**. (10 points) An egg is taken out of the refrigerator at $40°F$ and immediately put in a pot of boiling water at $212°F$. After $1$ minute, the temperature has risen to $126°F$. How long will it take for the egg to reach $169°F$?
NOTE: These numbers have nothing to do with the real world.
They are chosen to make two fractions simplify to $\frac{1}{2}$ and $\frac{1}{4}$.

**5**. (16 points) A sequence is defined by $a_1 = 4$ and $a_{n+1} = 2\sqrt{a_n} + 3$.

**a**. (2 pts) Find $a_2$ and $a_3$.

**b**. (4 pts) Assuming the limit exists, find the possible values of the limit.

**c**. (4 pts) Prove the sequence is increasing or decreasing.

**d**. (4 pts) Prove the sequence is bounded above (if increasing) or below (if decreasing).

**e**. (2 pts) State the theorem that shows the sequence converges and state the limit.

**6**. (12 points) Determine whether each series is convergent or divergent.
Name each Convergence Test(s) you use and check their assumptions. Use sentences!

**a**. (4 pts) $\sum_{n=0}^{\infty} \frac{5}{n^3 + \sqrt[3]{n}}$

**b**. (4 pts) $\sum_{n=0}^{\infty} \frac{e^n}{(1 + e^n)^2}$

**c**. (4 pts) $\sum_{n=2}^{\infty} \frac{n + \sqrt{n}}{n^2 + \sqrt{n}}$

**7**. (8 points) The series $S = \sum_{n=0}^{\infty} \frac{24n^2}{(n^3 + 1000)^3}$ is convergent by the Integral Test.

If the series is approximated by its $10^{\text{th}}$ partial sum $S_{10} = \sum_{n=0}^{10} \frac{24n^2}{(n^3 + 1000)^3}$,

find a bound on the error in this approximation: $E_{10} = S - S_{10}$.

**8**. (10 points) The series $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$ is: (Explain all reasoning. Circle your answer.)

**a**. Absolutely Convergent

**b**. Absolutely Divergent

**c**. Conditionally Convergent

**d**. Conditionally Divergent

**e**. Divergent

**9**. (12 points) A bucket contains $50$ gal of salt water containing $5$ lb of salt.

Salt water with a concentration of $0.2 \frac{\text{lb}}{\text{gal}}$ is added at the rate $5 \frac{\text{gal}}{\text{min}}$.

The water is kept well mixed and drained at $5 \frac{\text{gal}}{\text{min}}$.

How much salt is in the bucket after $10$ min?

**10**. (13 points)   Find the Interval of Convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n}\,3^n}(x-2)^n$.

Follow these steps. Be sure to name any Convergence Tests you use.

**a**. (4 pts)   Find the Radius of Convergence.

**b**. (4 pts)   Check convergence at the left endpoint.
Be sure to name and check out any Convergence Tests you use.
$x =$ ______________

**c**. (4 pts)   Check convergence at the right endpoint.
Be sure to name and check out any Convergence Tests you use.
$x =$ ______________

**d**. (1 pts)   State the Interval of Convergence.