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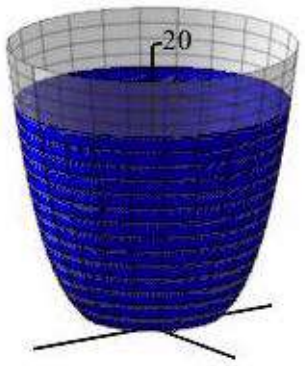
MATH 172H Quiz 1 Spring 2024

Sections 200 Solutions P. Yasskin

**1**. (8 points) A water tank is formed by rotating the curve $y = x^4$ for $y \le 20$ meters about the $y$-axis. It is filled to a depth of $16$ meters. Find the volume of water in the tank. Set up the integral using horizontal slices.
(Setting up the integral is 3 points.)




**Solution**: The slice at height $y$ is a circle of radius $r = x = y^{1/4}$.
So the cross sectional area is $A = \pi r^2 = \pi y^{1/2}$. And the volume is

$$V = \int_0^{16} A\,dy = \int_0^{16} \pi y^{1/2}\,dy = \pi\left[\frac{2y^{3/2}}{3}\right]_0^{16} = \frac{2\pi}{3}64 = \frac{128}{3}\pi$$

**2**. (12 points) A water tank is formed by rotating the curve $y = x^4$ for $y \le 20$ meters about the $y$-axis. It is filled to a depth of $16$ meters. Find the work done to pump the water out the top of the tank.
Answers can be given as a multiple of $\delta g$ where
$\delta$ is the densty of water and $g$ is the acceleration of gravity is $g$.
Don't simplify.

**Solution**: A horizontal slice at height $y$ has area $A = \pi r^2 = \pi y^{1/2}$.
Its volume is $dV = A\,dy = \pi y^{1/2}\,dy$. So its weight is $dF = \delta g\,dV = \delta g\,\pi y^{1/2}\,dy$.
It needs to be lifted a distance $D = 20 - y$. So the work done is

$$W = \int D\,dF = \int_0^{16} (20 - y)\delta g\pi y^{1/2}\,dy = \delta g\pi \int_0^{16} (20y^{1/2} - y^{3/2})\,dy$$
$$= \delta g\pi\left[20\frac{2y^{3/2}}{3} - \frac{2y^{5/2}}{5}\right]_0^{16} = \delta g\pi\left(20\frac{2}{3}16^{3/2} - \frac{2}{5}16^{5/2}\right)$$
$$= \delta g\pi\left(5\frac{2^3}{3}2^6 - \frac{2}{5}2^{10}\right) = \delta g\pi 2^9\left(\frac{5}{3} - \frac{4}{5}\right) = \delta g\pi 2^9\frac{13}{25}$$