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MATH 221 Exam 1, Version A Spring 2024

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1-8	/48	11	/12
9	/20	12	/12
10	/12	Total	/104

Multiple Choice: (6 points each. No part credit.)

1. A point has spherical coordinates $(\rho,\phi,\theta) = \left(6, \frac{\pi}{6}, \frac{\pi}{4}\right)$. Find its cylindrical coordinates.

a. $(r,\theta,z) = \left(3, \frac{\pi}{6}, 3\sqrt{3}\right)$

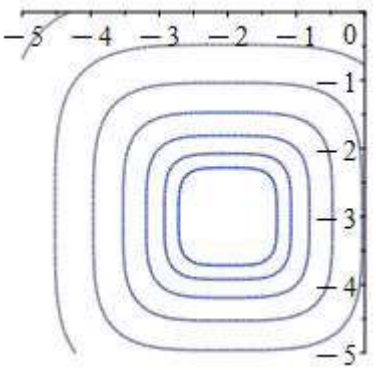
b. $(r,\theta,z) = \left(3, \frac{\pi}{4}, 3\sqrt{3}\right)$

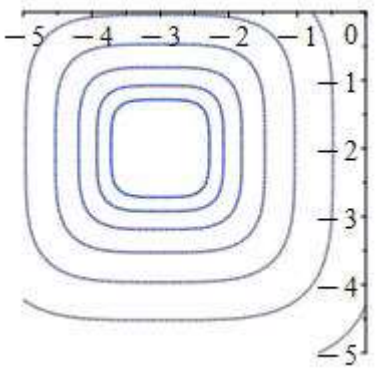
c. $(r,\theta,z) = \left(3, \frac{\pi}{6}, 6\sqrt{3}\right)$

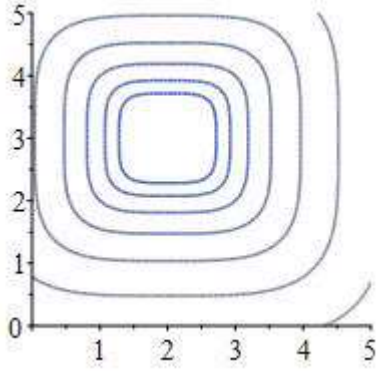
d. $(r,\theta,z) = \left(6\sqrt{3}, \frac{\pi}{6}, 3\right)$

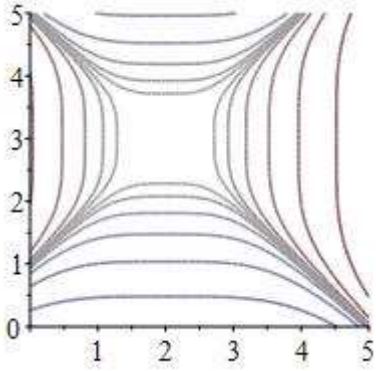
e. $(r,\theta,z) = \left(3\sqrt{3}, \frac{\pi}{4}, 3\right)$

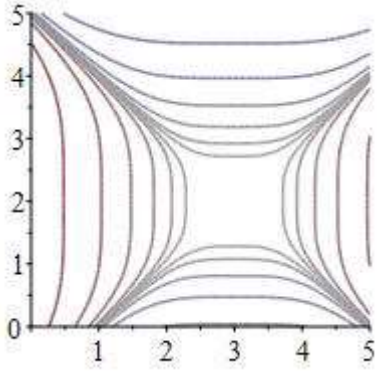
2. Which of the following is the contour plot of the function $f(x,y) = (x-2)^4 + (y-3)^4$?

a. 

b. 

c. 

d. 

e. 

3. A hiker starts at the point $P = (4,2)$, travels along the vector $\vec{a} = \langle 2,-2 \rangle$, then along the vector $\vec{b} = \langle 1,3 \rangle$ and finally along the vector $\vec{c} = \langle -1,2 \rangle$. Along what vector should the hiker travel to get back to the starting point P?

a. $\langle -2,-3 \rangle$
b. $\langle -6,-5 \rangle$
c. $\langle 6,5 \rangle$
d. $\langle 2,3 \rangle$
e. $\langle 2,-1 \rangle$

4. For what value of p is $\vec{u} = \langle p,5,3 \rangle$ perpendicular to $\vec{v} = \langle 2,1,p \rangle$?

a. $p = -2$
b. $p = -1$
c. $p = 0$
d. $p = 1$
e. $p = 2$

5. Find the volume of the parallelepiped with edge vectors $\vec{a} = \langle 4,2,0 \rangle$, $\vec{b} = \langle 1,0,-3 \rangle$ and $\vec{c} = \langle 0,-1,2 \rangle$.

a. -16
b. -12
c. 8
d. 12
e. 16

6. If $\hat{T}$ points Up and $\hat{B}$ points NorthEast, in what direction does $\hat{N}$ point?

a. SouthEast
b. SouthWest
c. NorthWest
d. Down

7. Which of the following is a plane perpendicular to the line $(x,y,z) = (1+3t, 3+2t, 4-t)$?

a. $3x - 2y - z = 3$
b. $-3x + 2y + z = 2$
c. $x + 3y + 4z = 5$
d. $3x + 2y - z = 7$
e. $x - 3y + 4z = 5$

8. Classify the quadratic surface: $-x^2 + 2x + y^2 + 4y - 2z^2 + 12z = 14$

a. Hyperbolic Paraboloid opening up in the x-direction and down in the y-direction
b. Hyperbolic Paraboloid opening up in the y-direction and down in the x-direction
c. Hyperboloid of 1 sheet
d. Hyperboloid of 2 sheets
e. Cone

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (20 pts) Consider the twisted cubic $\vec{r} = (t^3, 3t^2, 6t)$. Compute each of the following.
Note: $t^4 + 4t^2 + 4 = (t^2 + 2)^2$

a. (6 pts) Arc length between $(0,0,0)$ and $(1,3,6)$.

b. (6 pts) Curvature $\kappa = \dfrac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$.
HINT: Factor out an 18^2.

c. (4 pts) Tangential acceleration, a_T.
HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

d. (4 pts) Normal acceleration, a_N.
HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

10. (12 pts) Write the vector, $\vec{a} = \langle 5, -3, 1 \rangle$, as a sum of two vectors $\vec{p}$ and $\vec{q}$, where $\vec{p}$ is parallel to $\vec{b} = \langle 6, 2, 4 \rangle$ and $\vec{q}$ is perpendicular to $\vec{b}$.

11. (12 pts) Consider the helix $\vec{r}(\theta) = \langle 4\cos\theta, 4\sin\theta, 3\theta \rangle$ for $0 \le \theta \le 2\pi$.

a. Find its mass, if its linear density is $\delta(x, y, z) = z$.

b. Find the work done to push a bead along the helix if the force is $\vec{F} = \langle -2y, 2x, 0 \rangle$.

12. (12 pts) Consider the planes:

$$P_1: \quad x + y - z = 3$$
$$P_2: \quad x + 3y + 3z = 5$$

Determine if they are parallel or intersecting. If they intersect, find the line of intersection. You MUST explain why they are or are not parallel.