Name______________________________

MATH 221 Exam 2, Version A Spring 2024

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1-9	/81	11	/12
10	/12	12	/105

Multiple Choice: (9 points each. No part credit.)
Circle you answers here and bubble on the Scantron.
Show your work, in case I give some part credit.

1. Find the plane tangent to the surface $z = xy^2 + x^3 y$ at $(x,y) = (1,2)$.
What is the z-intercept?

a. $c = -14$
b. $c = -6$
c. $c = 0$
d. $c = 6$
e. $c = 14$

2. Find the plane tangent to the surface $xyz^2 = 6$ at $(x,y,z) = (3,2,1)$.
What is the z-intercept?

a. $c = -12$
b. $c = -2$
c. $c = 0$
d. $c = 2$
e. $c = 12$

3. A weather balloon measures the temperature and its gradient at $P = (3,4,2)$ to be:

$$T = 70^\circ \qquad \vec{\nabla} T|_P = \langle -3,2,2 \rangle$$

Approximate the temperature at $(x,y,z) = (3.2,3.7,2.2)$.

a. 68.4°
b. 69.2°
c. 70.2°
d. 70.8°
e. 71.2°

4. A weather balloon measures the temperature and its gradient at $P = (3,4,2)$ to be:

$$T = 70^\circ \qquad \vec{\nabla} T|_P = \langle -3,2,2 \rangle$$

If the balloon's velocity is $\vec{v} = \langle 2,4,-2 \rangle$, how fast is the temperature changing as seen aboard the balloon? $\dfrac{dT}{dt} =$

a. -2
b. -1
c. 0
d. 1
e. 2

5. The equation $x^2 z + yz^2 = z^3 + 6$ implicitly defines z as a function of (x,y) near the point $P = (x,y,z) = (2,3,1)$. Find $\left.\dfrac{\partial z}{\partial y}\right|_P$.

a. $\dfrac{\partial z}{\partial y} = \dfrac{-1}{13}$
b. $\dfrac{\partial z}{\partial y} = \dfrac{-1}{\sqrt[3]{6}}$
c. $\dfrac{\partial z}{\partial y} = \dfrac{-1}{7}$
d. $\dfrac{\partial z}{\partial y} = \dfrac{1}{7}$
e. $\dfrac{\partial z}{\partial y} = \dfrac{1}{13}$

6. Two marbles are located at $P=(a,b)$ and $X=(x,y)$. Their current positions and velocities are:

$$P=(1,2) \qquad X=(5,5) \qquad \frac{dP}{dt}=\langle 15,-10\rangle \qquad \frac{dX}{dt}=\langle 5,15\rangle$$

How fast is the distance between them changing?
HINT: There are 4 intermediate variables.

a. $\dfrac{dD}{dt}=-1$
b. $\dfrac{dD}{dt}=1$
c. $\dfrac{dD}{dt}=3$
d. $\dfrac{dD}{dt}=5$
e. $\dfrac{dD}{dt}=7$

7. Queen Lena is flying the Centurian Eagle through the Force whose density is $F=x^3y^2z\,\dfrac{\text{yodons}}{\text{lightsec}^3}$.

If she is located at $(x,y,z)=(1,2,3)$ and travels in the direction of maximum increase of the Force with speed $|\vec{v}|=3\dfrac{\text{lightsec}}{\text{lightsec}}$, what is the rate she sees the Force increasing?

a. $\dfrac{dF}{dt}=3\sqrt{91}$
b. $\dfrac{dF}{dt}=4\sqrt{91}$
c. $\dfrac{dF}{dt}=6\sqrt{91}$
d. $\dfrac{dF}{dt}=12\sqrt{91}$
e. $\dfrac{dF}{dt}=48\sqrt{91}$

8. If $f(x,y) = x^2 \cos(xy)$ which of the following is FALSE?

a. $f_{xx} = 2\cos(xy) - 4xy\sin(xy) - x^2y^2\cos(xy)$

b. $f_{xy} = -3x^2\sin(xy) - x^3y\cos(xy)$

c. $f_{yx} = -3x^2\cos xy + x^3y\sin xy$

d. $f_{yy} = -x^4\cos(xy)$

9. The point $(2,-2)$ is a critical point of the function $f = y^3 - x^3 - 6xy$.
Classify this critical point using the Second Derivative Test.

a. Local Minimum

b. Local Maximum

c. Inflection Point

d. Saddle Point

e. Test Fails

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (12 pts) If the limit converges, prove it and find the limit.
If it diverges , give 2 curves which give different limits.

a. (6 pts) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$

b. (6 pts) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2 y^3}{(x^2 + y^2)^2}$

11. (12 pts) Find the volume of the cylindrical can with the largest volume, if its surface area is $A = 24\pi$.
HINT: The surface area is $A = 2\pi rh + 2\pi r^2$.