## Name

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MATH 221

Prove the Pythagorean Identity for Vectors:

$$
(\vec{u} \cdot \vec{v})^{2}+|\vec{u} \times \vec{v}|^{2}=|\vec{u}|^{2}|\vec{v}|^{2}
$$

as follows: Consider the vectors $\vec{u}=\left(u_{1}, u_{2}, u_{3}\right)$ and $\vec{v}=\left(v_{1}, v_{2}, v_{3}\right)$.
Compute each of the following by hand on paper. Show your work. Simplify where possible.

1. Write the algebraic definition for $\vec{u} \cdot \vec{v}$.
$\vec{u} \cdot \vec{v}=$
2. Write out $(\vec{u} \cdot \vec{v})^{2}$ to get 6 terms. $(\vec{u} \cdot \vec{v})^{2}=$
3. Write the algebraic definition for $\vec{u} \times \vec{v}$.
$\vec{u} \times \vec{v}=$
4. Write out $|\vec{u} \times \vec{v}|^{2}$ to get 9 terms. $|\vec{u} \times \vec{v}|^{2}=$
5. Add $(\vec{u} \cdot \vec{v})^{2}+|\vec{u} \times \vec{v}|^{2} \quad$ and cancel some terms. $(\vec{u} \cdot \vec{v})^{2}+|\vec{u} \times \vec{v}|^{2}=$
6. Multiply out $|\vec{u}|^{2}|\vec{v}|^{2}$.
$|\vec{u}|^{2}|\vec{v}|^{2}=$
7. Are the answers to (5) and (6) equal?
