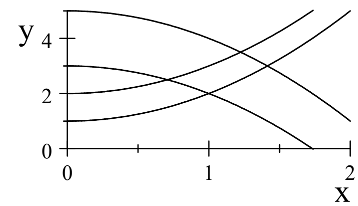
Name________________________ Section________________________

MATH 221 Take Home Quiz 3 P. Yasskin

1. Compute the integral $\iint x\,dA$ over the region in the first quadrant bounded by $y = 1 + x^2$, $y = 2 + x^2$, $y = 3 - x^2$, and $y = 5 - x^2$.



a. Define the curvilinear coordinates u and v by $y = u + x^2$ and $y = v - x^2$.
What are the 4 boundaries in terms of u and v?

b. Solve for x and y in terms of u and v. Express the results as a position vector.

$\vec{R}(u,v) = (x(u,v), y(u,v)) =$

c. Find the coordinate tangent vectors:

$\vec{e}_u = \dfrac{\partial\vec{R}}{\partial u} =$

$\vec{e}_v = \dfrac{\partial\vec{R}}{\partial v} =$

d. Compute the Jacobian determinant:

$\dfrac{\partial(x,y)}{\partial(u,v)} =$

e. Compute the Jacobian factor:

$J = \left|\dfrac{\partial(x,y)}{\partial(u,v)}\right| =$

f. Compute the integral:

$\iint x\,dA =$

2. Find the Jacobian for spherical coordinates. The position vector is given by

$$\vec{R}(\rho,\theta,\varphi) = (\rho\sin\varphi\cos\theta, \rho\sin\varphi\sin\theta, \rho\cos\varphi)$$

a. Find the coordinate tangent vectors:

$$\vec{e}_\rho = \frac{\partial\vec{R}}{\partial\rho} =$$

$$\vec{e}_\theta = \frac{\partial\vec{R}}{\partial\theta} =$$

$$\vec{e}_\varphi = \frac{\partial\vec{R}}{\partial\varphi} =$$

b. Compute the Jacobian determinant:

$$\frac{\partial(x,y,z)}{\partial(\rho,\theta,\varphi)} =$$

c. Compute the Jacobian factor:

$$J = \left|\frac{\partial(x,y,z)}{\partial(\rho,\theta,\varphi)}\right| =$$