

**MATH 151, FALL SEMESTER 2004
COMMON EXAMINATION I - VERSION A**

Name (print): _____

Signature: _____

Instructor's name: _____

Section No: _____

Seat No: _____

INSTRUCTIONS

1. In Part 1 (Problems 1–13), mark the correct choice on your ScanTron form using a No. 2 pencil. *For your own record, also mark your choices on the exam.* ScanTrons will be collected from all examinees after one hour, and will *not* be returned.
2. Calculators may *not* be used in Part 1. The use of calculators is permitted *only after the first hour has elapsed and all ScanTrons have been collected.*
3. In Part 2 (Problems 14–18), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form.**

<u>QN</u>	<u>PTS</u>
<u>1–13</u>	_____
<u>14</u>	_____
<u>15</u>	_____
<u>16</u>	_____
<u>17</u>	_____
<u>18</u>	_____
TOTAL	

Part 1 – Multiple Choice (52 points)

Read each question carefully; each problem is worth **4 points**. Calculators are **not** allowed for this part of the exam.

1. If $\mathbf{a} = \langle 2, 3 \rangle$ and $\mathbf{b} = \langle 1, -1 \rangle$, find $2\mathbf{a} - 3\mathbf{b}$.
 - (a) $\langle 1, 9 \rangle$
 - (b) $\langle 4, 11 \rangle$
 - (c) $\langle 1, 3 \rangle$
 - (d) $\langle 4, 7 \rangle$
 - (e) $\langle 1, 1 \rangle$
2. Let A , B , and C be the vertices of a triangle. If $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$, what is \overrightarrow{BC} ?
 - (a) $\mathbf{a} + \mathbf{b}$
 - (b) $\mathbf{a} - \mathbf{b}$
 - (c) $\mathbf{b} - \mathbf{a}$
 - (d) $\mathbf{a} \cdot \mathbf{b}$
 - (e) $\frac{1}{2}(\mathbf{a} + \mathbf{b})$
3. Let $\mathbf{a} = \langle -2, 3 \rangle$ and $\mathbf{b} = \langle 1, 2 \rangle$. Find $\text{comp}_{\mathbf{a}}\mathbf{b}$, *i.e.*, the scalar projection of \mathbf{b} onto \mathbf{a} .
 - (a) $\sqrt{13}/4$
 - (b) $8/\sqrt{13}$
 - (c) $\sqrt{13}/8$
 - (d) $4/\sqrt{13}$
 - (e) $4/13$
4. Suppose that \mathbf{u} and \mathbf{v} are unit vectors which subtend an angle of 60 degrees between them. Compute the dot product $\mathbf{v} \cdot (2\mathbf{u} - 3\mathbf{v})$.
 - (a) -2
 - (b) -1
 - (c) 0
 - (d) 1
 - (e) 2

5. The parametric curve determined by the equations $x(t) = \sin t$, $y(t) = \cos^2 t$, $0 \leq t \leq \pi/2$, forms a:
- (a) part of a parabola
 - (b) part of a hyperbola
 - (c) part of a circle
 - (d) line segment
 - (e) none of the above
6. Suppose that f is a function defined on an open interval containing the point 2, and that $f(2) = 3$. Which of the following statements is always true?
- (a) $\lim_{x \rightarrow 2} f(x) = 3$.
 - (b) Either $\lim_{x \rightarrow 2^-} f(x)$ or $\lim_{x \rightarrow 2^+} f(x)$ must exist.
 - (c) If $\lim_{x \rightarrow 2} f(x)$ exists, then f is continuous at $x = 2$.
 - (d) If $\lim_{x \rightarrow 2} f(x) = 3$, then f is continuous at $x = 2$.
 - (e) If $\lim_{x \rightarrow 2} f(x) = 3$, then f is differentiable at $x = 2$.
7. Which of the following intervals contains a number c satisfying the equation $c^3 + c - 1 = \pi^2$?
- (a) $(0, 1)$
 - (b) $(1, 2)$
 - (c) $(2, 3)$
 - (d) $(3, \infty)$
 - (e) none of the above
8. Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt{2x^4 + 3x^2 + 1}}{3x^2 + 2x + 4}$.
- (a) $2/3$
 - (b) $\sqrt{3}/2$
 - (c) $\sqrt{2}/3$
 - (d) $3/\sqrt{2}$
 - (e) $1/4$
9. Find all the vertical asymptotes of the curve $y = \frac{x - 2}{x^2 - 4}$.
- (a) $x = \pm 2$
 - (b) $x = -2$ only
 - (c) $x = 2$ only
 - (d) $x = 0$
 - (e) there are no vertical asymptotes

10. Let $f(x) = |2x - 3|$. Find the value(s) of x for which $f'(x)$ does not exist.
- (a) $x = \pm 3/2$
 - (b) $x = 2/3$
 - (c) $x = 3/2$
 - (d) $x = 0, 3/2$
 - (e) $x = 0, 2/3$
11. Differentiate the function $f(x) = \frac{2x - 1}{x^2 + 1}$ with respect to x .
- (a) $\frac{2(1 + x - x^2)}{x^2 + 1}$
 - (b) $\frac{2(1 + x - x^2)}{(x^2 + 1)^2}$
 - (c) $\frac{2(x^2 - x - 1)}{(x^2 + 1)^2}$
 - (d) $\frac{1}{x}$
 - (e) $-\frac{2}{x^2}$
12. A particle is moving along a straight line and its position function is given by $s(t) = t^2 + t + 1$, where t is measured in seconds and s is measured in meters. Find the velocity of the particle (in meters per second) after 2 seconds.
- (a) 2
 - (b) 5
 - (c) 3
 - (d) 7
 - (e) 0
13. Suppose that $f(0) = 0$ and $\lim_{x \rightarrow 0} \frac{f(x)}{x} = -1$. If $g(x) = (2x - 1)f(x)$, find $g'(0)$. (Hint: What is $f'(0)$?)
- (a) 1
 - (b) -1
 - (c) 0
 - (d) -2
 - (e) 2

Part 2 (52 points, including a bonus of 4)

Calculators are allowed for this part of the exam. Refer to the front page for further instructions.

- 14. (8 points)** A woman walks due west on the deck of a ship at 3 miles per hour. The ship is sailing north at a speed of 22 miles an hour. Find the speed and direction (expressed as an angle θ) of the woman relative to the surface of the water. Draw a figure and label θ .

- 15. (8 points)** Obtain a vector equation of the straight line passing through the point $(3, 2)$ and perpendicular to the vector joining the points $(1, 1)$ and $(2, -1)$.

16. (9 points) Let c be a fixed real number and let f be defined as follows:

$$f(x) = \begin{cases} 2c^2x^2 + cx + c, & \text{if } x < 1; \\ 1, & \text{if } x = 1; \\ cx + 1, & \text{if } x > 1. \end{cases}$$

Find all possible values of c for which $\lim_{x \rightarrow 1} f(x)$ exists. Explain your reasoning carefully and concisely.

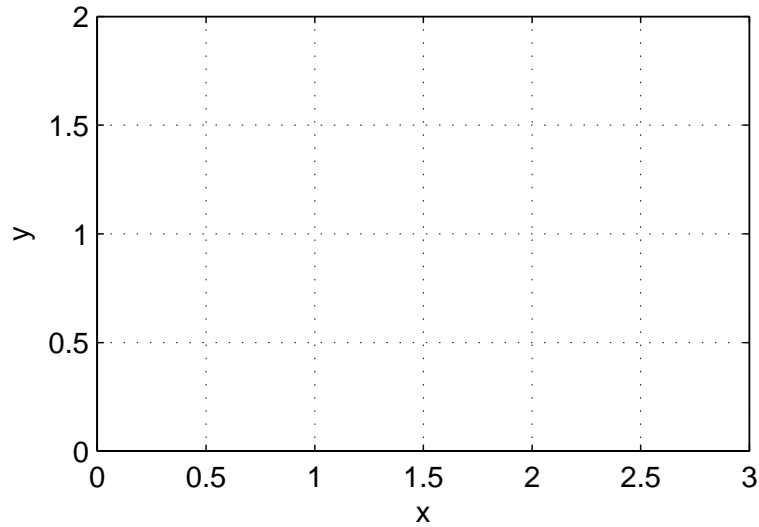
17. (i) (3 points) Give a precise definition of the derivative of a function f at a point a .

(ii) (7 points) Let $f(x) = \frac{1}{2x+3}$. Use the **definition of the derivative** to compute $f'(1)$.
(Note: No credit will be given for using any other method, correct answer notwithstanding.)

(iii) (3 points) Obtain an equation of the tangent to the curve $y = \frac{1}{2x+3}$ when $x = 1$.

18. Consider the functions f and g defined on the closed interval $[0, 3]$ as follows: the graph of f consists of a straight-line segment from the point $(0, 1)$ to the point $(1, 1)$, followed by a line segment from $(1, 1)$ to $(3, 2)$. The graph of g comprises a line segment from the point $(0, 2)$ to the point $(3/2, 0)$, followed by a line segment from $(3/2, 0)$ to $(3, 1)$.

(i) (2 points) Sketch the graphs of f and g in the grid provided below.



(ii) (1 point) At which value(s) of x in the open interval $(0, 3)$ is f not differentiable?

(iii) (1 point) At which value(s) x in the open interval $(0, 3)$ is g not differentiable?

(iv) (5 points) Find the derivative of fg at the point $x = 1/2$.

(v) (5 points) Find the derivative of $\frac{f}{f+g}$ at the point $x = 2$.