

**MATH 151, FALL SEMESTER 2004  
COMMON EXAMINATION III - VERSION B**

Name (print): \_\_\_\_\_

Signature: \_\_\_\_\_

Instructor's name: \_\_\_\_\_

Section No: \_\_\_\_\_

Seat No: \_\_\_\_\_

**INSTRUCTIONS**

1. In Part 1 (Problems 1–13), mark the correct choice on your ScanTron form using a No:2 pencil. For your own record, mark your choices on the exam itself. ScanTrons will be collected from all examinees after one hour, and will not be returned. No student will be allowed to turn in his/her scantron before the end of the first hour.
2. Calculators may not be used in Part 1. The use of calculators is permitted only after the first hour has elapsed and all ScanTrons have been collected.
3. In Part 2 (Problems 14–18), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly.** You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form.**

<u>QN</u>	<u>PTS</u>
1–13	_____
14	_____
15	_____
16	_____
17	_____
18	_____
<b>TOTAL</b>	

**Part 1 – Multiple Choice (52 points)**

Read each question carefully; each problem is worth **4 points**. Calculators are **not** allowed for this part of the exam.

1. Differentiate the function  $f(x) = \ln(2x + 4)$  with respect to  $x$ .

(a)  $e^{2x+4}$

(b)  $\frac{1}{x+2}$

(c)  $\frac{1}{2x+4}$

(d)  $\frac{1}{x}$

(e)  $\frac{1}{2x}$

2. Evaluate  $\lim_{x \rightarrow 0} \frac{e^{x^2} - 1 - x^2}{x^3}$ .

(a)  $-\infty$

(b)  $-\frac{1}{2}$

(c)  $\frac{1}{2}$

(d) 0

(e)  $+\infty$

3. Find  $\lim_{x \rightarrow 0^+} \frac{\ln x}{\sqrt{x}}$ .

(a)  $+\infty$

(b)  $-\infty$

(c)  $-1$

(d) 0

(e) 1

4. Evaluate  $\cos^{-1}(\cos(3\pi))$ .

(a)  $-\pi$

(b)  $\pi$

(c)  $2\pi$

(d)  $3\pi$

(e)  $\frac{\pi}{2}$

5. Compute  $\cos(2 \tan^{-1}(3))$ . (Note:  $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$ .)

(a)  $-\frac{1}{5}$

(b)  $-\frac{4}{5}$

(c)  $-\frac{2}{\sqrt{10}}$

(d)  $\frac{1}{5}$

(e)  $\frac{4}{5}$

6. Find an antiderivative of the function  $f(x) = \frac{x^2 + 1}{x^2}$  in the interval  $(0, \infty)$ .

(a)  $\frac{(x^3/3) + x}{(x^3/3)}$

(b)  $\frac{x^2(x^3/3 + x) - (x^2 + 1)(x^3/3)}{x^4}$

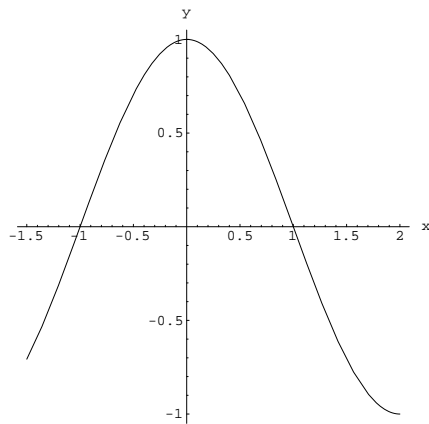
(c)  $x - 2x^{-3}$

(d)  $\frac{x^2 + 1}{x}$

(e)  $\frac{x^2 - 1}{x}$

7. If  $F$  is an antiderivative of  $f$ , which of the following functions is an antiderivative of  $s(x) = f(3x)$ ?
- (a)  $F(3x)$
  - (b)  $3F(3x)$
  - (c)  $\frac{F(3x)}{3}$
  - (d)  $F'(3x)$
  - (e) cannot be determined with the information given
8. A particle moves in a straight line and has acceleration given by  $a(t) = 6t$ . Its initial velocity is  $v(0) = 1$  cm/sec and its initial position is  $s(0) = 1$  cm. Find  $s(1)$ .
- (a) 0
  - (b) 6
  - (c) 4
  - (d) 3
  - (e) 1
9. At which point in the closed interval  $[-1, 1]$  does the function  $f(x) = x - \sin^{-1}(x)$  attain its absolute maximum value?
- (a)  $x = 0$
  - (b)  $x = 1$
  - (c)  $x = \frac{1}{2}$
  - (d)  $x = -1$
  - (e)  $f$  does not attain an absolute maximum value in the said interval

Suppose that  $H$  is a differentiable function. Given below is the graph of the **derivative** of  $H$ . Use it to answer the next two questions (Questions 10 and 11).



10. On which of the following intervals is  $H$  increasing?
- (a)  $(-\frac{3}{2}, 1)$
  - (b)  $(-1, 2)$
  - (c)  $(-\frac{3}{2}, 0)$
  - (d)  $(-1, 1)$
  - (e)  $(-\frac{3}{2}, 2)$
11. At which of the following points does  $H$  attain a local minimum?
- (a)  $x = -1$
  - (b)  $x = \frac{3}{2}$
  - (c)  $x = 1$
  - (d)  $x = 0$
  - (e)  $H$  does not attain a local minimum in the interval  $(-\frac{3}{2}, 2)$

**12.** Which of the following points is a point of inflection on the graph of  $y = xe^x$ ?

(a)  $(-1, -e^{-1})$

(b)  $(0, 0)$

(c)  $(2, 2e^2)$

(d)  $(-2, -2e^{-2})$

(e) the graph has no points of inflection

**13.** Find all the critical numbers of the function  $f(x) = x^2 - |2x - 1|$ .

(a)  $x = 0$

(b)  $x = \frac{1}{2}$

(c)  $x = -1, 1$

(d)  $x = -1, \frac{1}{2}, 1$

(e)  $x = -1, 0, 1$

**Part 2 (54 points)**

*The use of a calculator is permitted for this part of the exam, but all work must be shown in order to receive credit. Refer to the front page for further instructions.*

- 14. (8 points)** A bacterial culture starts with 1500 bacteria, and after 2 hours the population is 2800 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after 6 hours.

- 15. (12 points)** Suppose that  $f$  is a differentiable function. Find the derivative of each of the following functions. Your answer may be in terms of  $f$  and/or  $f'$ .

(i) (6 points)  $U(x) = f(\tan^{-1}(e^{-x}))$

(i) (6 points)  $V(x) = (2 + \cos x)^{f(x)}$

16. (10 points) Evaluate

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{3x}\right)^{2x}.$$

Show all your steps clearly and methodically.



17. (12 points) Suppose that  $f$  is twice differentiable on  $(-\infty, \infty)$ , and that

$$f'(x) = \frac{-4x}{(x^2 + 3)^2} \quad \text{and} \quad f''(x) = \frac{12x^2 - 12}{(x^2 + 3)^3}, \quad -\infty < x < \infty.$$

(i) (2 points) Find the interval(s) in which  $f$  is increasing.

(ii) (4 points) Find the interval(s) in which the graph of  $f$  is concave up.

(iii) (4 points) Find, if any, the  $x$ -values at which  $f$  attains its local maximum, local minimum, absolute maximum, and absolute minimum. If any of these do not exist, say so.

(iv) (2 points) Find, if any, the  $x$ -values corresponding to which the graph of  $f$  has a point of inflection.

**18. (12 points)** A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus, the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 30 feet, find the dimensions of the window that maximize its area.

(*Note:* The circumference of a semicircle is  $\pi$  times the radius of the circle.)

(i) (2 points) Draw a figure and label the relevant variables.

(ii) (5 points) Find a function (of one variable) which is to be maximized, and state the domain in which it is to be maximized.

(iii) (5 points) Complete the problem. Show your final answer clearly.