MATH 151 Regular, **FALL SEMESTER 2011**
FINAL EXAMINATION

Name (print): ______________________________ Instructor's name: Yasskin

Signature: ______________________________ Section No: __________

Part 1 – Multiple Choice (**15 questions**, **4 points each**, **No Calculators**)

Write your name and section number on the ScanTron form.
Mark your responses on the ScanTron form and on the exam itself

1. Evaluate $\lim\limits_{x\to-1} \dfrac{x^3+9x^2+8x}{x^2-1}$

a. 6
b. ∞
c. $\dfrac{7}{2}$
d. 0
e. 3

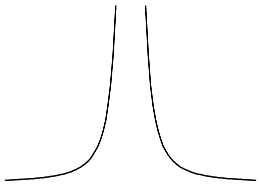
2. The limit $\lim\limits_{h\to0} \dfrac{4(2+h)^4-64}{h}$ can be interpreted as which of the following?

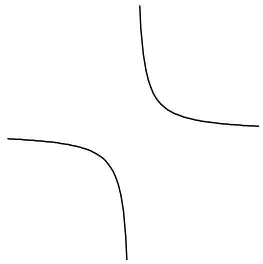
a. $f'(64)$ where $f(x)=4x^4$
b. $f'(4)$ where $f(x)=x^4$
c. $f'(2)$ where $f(x)=16x^3$
d. $f'(2)$ where $f(x)=\frac{4}{5}x^5$
e. $f'(2)$ where $f(x)=4x^4$

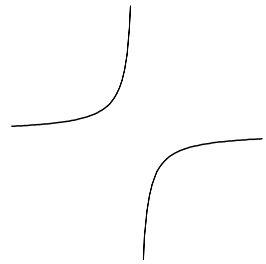
3. Find the line tangent to $y=\sin x$ at $x=\dfrac{\pi}{3}$. Its y-intercept is

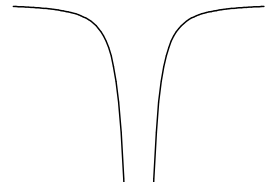
a. $\dfrac{\sqrt{3}}{2}+\dfrac{\pi}{6}$
b. $\dfrac{\sqrt{3}}{2}-\dfrac{\pi}{6}$
c. $\dfrac{1}{2}-\dfrac{\sqrt{3}\,\pi}{6}$
d. $\dfrac{1}{2}+\dfrac{\sqrt{3}\,\pi}{6}$
e. 0

4. The function $f(x) = \dfrac{x^2 - 4x + 3}{x^2 - 4x + 4}$ has a vertical asymptote at $x = 2$. Near $x = 2$, the graph has the shape:

a. 

b. 

c. 

d. 

5. Find the absolute minimum value of $f(x) = 3x^2 - x^3$ on the interval $[1, 3]$.

a. 0
b. 2
c. 4
d. -2
e. -25

6. A spacecraft is being sent to Mars. Its distance from the earth is given by $p(t) = 7t^3 + 1$. At time $t = 2$ the position is measured, but the error in the time measurement is ± 0.1. What is the resulting error in the calculated position?

a. ± 7.3
b. ± 8.4
c. ± 8.5
d. ± 7.4
e. Impossible to determine.

7. Find two numbers a and b whose sum is 15 for which $P = a^2 b$ is a maximum. For this a and b we have $ab =$

a. 36
b. 44
c. 50
d. 54
e. 56

8. Let $f(x)$ be a differentiable function, and suppose $f(5) = 3$ and $f'(x) \leq 11$ for all values of x. Use the Mean Value Theorem to determine how large $f(7)$ can possibly be.

a. 25
b. -25
c. 19
d. 33
e. Not enough information.

9. Evaluate $\lim\limits_{x \to -\infty} \dfrac{\sqrt{9x^2 - 3}}{2x + 5}$

a. $\frac{-3}{4}$
b. $\frac{3}{4}$
c. $\frac{-3}{2}$
d. $\frac{3}{2}$
e. ∞

10. Approximate the area under $y = x^2 + 2$ above the x-axis between $x = 0$ and $x = 6$ using 3 intervals of equal length and rectangles whose heights are computed at the midpoints of each interval.

HINT: Draw a picture

a. 41
b. 52
c. 82
d. 84
e. 124

11. Evaluate $\displaystyle\int_{\pi/6}^{\pi/3} \sin(2x)\,dx$

a. $\frac{1}{2}$
b. $-\frac{1}{2}$
c. 1
d. -1
e. 0

12. If $I = \int_0^2 \sqrt{x^3+1}\,dx$, which of the following is FALSE?

a. $\int_0^5 \sqrt{x^3+1}\,dx - \int_2^5 \sqrt{x^3+1}\,dx = I$
b. $\int_0^2 x^3 + 1\,dx = I^2$
c. $\int_2^0 \sqrt{x^3+1}\,dx = -I$
d. $2 \le I \le 6$
e. $\int_0^2 2\sqrt{x^3+1}\,dx = 2I$

13. Calculate $\lim\limits_{x \to 0^+} \frac{1}{x} \int_0^x e^{t^2}\, dt$

a. $2e$
b. ∞
c. 0
d. 1
e. e

14. Calculate $\frac{d}{dx}\left(\frac{\ln(x)}{2^x}\right)$ at $x = 2$

a. $\frac{1}{8} - \frac{1}{4}\ln 2$
b. $\frac{1}{8} + \frac{1}{4}\ln 2$
c. $\frac{1}{8} - \frac{1}{4}(\ln 2)^2$
d. $\frac{1}{8} + \frac{1}{4}(\ln 2)^2$
e. $\frac{1}{8} - \frac{1}{4\ln 2}$

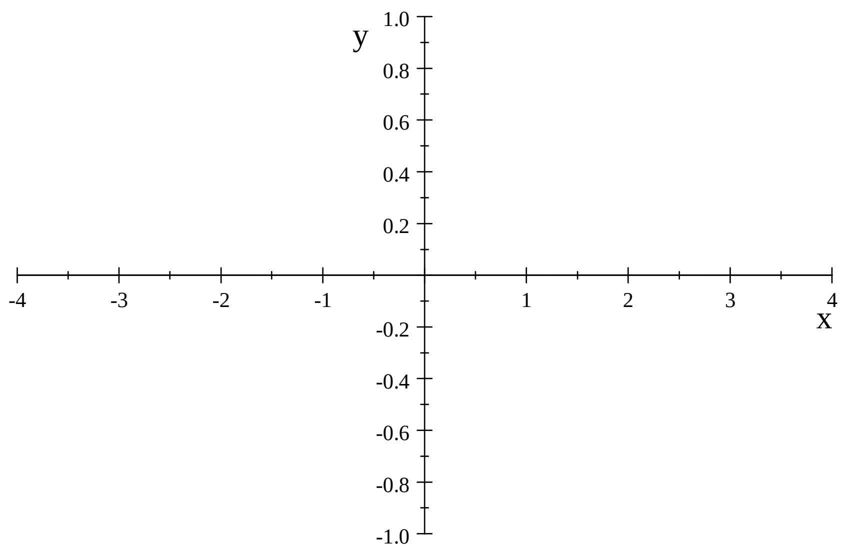
15. Find the intervals of concavity of the function $f(x) = 3(3 + x^2)^{-1}$.

a. Concave up: $(-1, 1)$ Concave down: $(-\infty, -1) \cup (1, \infty)$
b. Concave up: $(-\infty, \infty)$ Concave down: nowhere
c. Concave up: nowhere Concave down: $(-\infty, \infty)$
d. Concave up: $(1, \infty)$ Concave down: $(-\infty, -1) \cup (-1, 1)$
e. Concave up: $(-\infty, -1) \cup (1, \infty)$ Concave down: $(-1, 1)$

Part 2 – Work Out Problems (5 questions. Points indicated. No Calculators)

Solve each problem in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (6 points) Graph the function $f(x) = 3(3 + x^2)^{-1}$. (See problem #15.)
Be sure to label all maxima, minima, inflection points and asymptotes.
Be careful with intervals of increase, decrease and concavity.

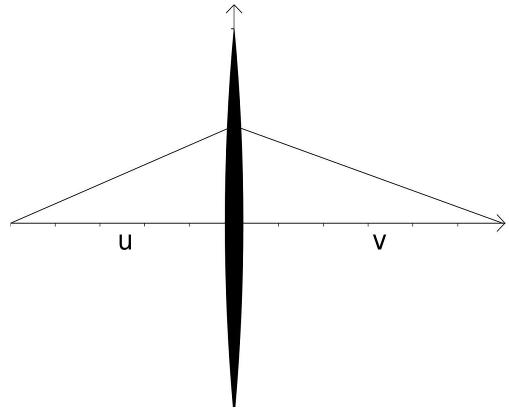


17. (8 points) Let $g(x)$ be the inverse function of $f(x) = xe^x$ for $x > 0$. Find $g(e)$ and $g'(e)$.

18. (10 points) When light passes through a lens with focal length f
the distance to the object, u, is related to
the distance to the image, v, by the equation

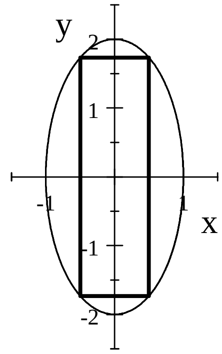
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

If $u = 2$, $v = 3$ and $\frac{du}{dt} = -0.4$, find f and $\frac{dv}{dt}$.
Is v getting longer or shorter?



19. (6 points) Find a parametric equation for the line tangent to the parametric curve $\vec{r}(t) = \langle t^2, t^3 \rangle$ at the point $(4, 8)$.

20. (12 points) A rectangle is inscribed in the ellipse $x^2 + \dfrac{y^2}{4} = 1$ with all sides parallel to the axes. Find the maximum area of such a rectangle.



Question	Points/Max
1-15	/60
16	/ 6
17	/ 8
18	/10
19	/ 6
20	/12
Total	/102

Name (print): ______________________ Section No: ______