Student (print) ______________________________ Section ____________

Last First Middle

Student (sign) ______________________________

Student ID ______________________________

Instructor ______________________________

# MATH 152
# Exam 3
# Fall 2001
# Test Form B

| | |
|---|---|
| 1–10 | /50 |
| 11 | /20 |
| 12 | /10 |
| 13 | /10 |
| 14 | /10 |
| total | |

Part I is multiple choice. There is no partial credit.
You may not use a calculator.

Part II is work out. Partial credit will be given.
You may use a calculator that cannot hold formulas.

## Part I: Multiple Choice (5 points each)
**There is no partial credit. You may not use a calculator.**

1. Find a unit vector perpendicular to $\vec{u} = \langle 4, 1, -1 \rangle$ and $\vec{v} = \langle 0, 1, 1 \rangle$.

   **A:** $\langle 0, 0, 1 \rangle$ **B:** $\left\langle \frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \right\rangle$ **C:** $\langle 1, -2, 2 \rangle$ **D:** $\left\langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \right\rangle$ **E:** $\left\langle \frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}} \right\rangle$

2. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}}{4^n}$.

   **A:** $\frac{4}{3}$ **B:** $6$ **C:** $\frac{3}{4}$ **D:** The series diverges **E:** $12$

3. Let $\{s_n\}_{n=1}^{\infty}$ be the sequence of partial sums of the series $\sum_{n=1}^{\infty} a_n$. If $s_n = \dfrac{n}{n+1}$ for $n = 1, 2, 3, \ldots$, find $a_3$.

**A:** $\dfrac{3}{4}$ **B:** $\dfrac{1}{6}$ **C:** $\dfrac{2}{3}$ **D:** $\dfrac{1}{12}$ **E:** $1$

4. For which values of $p$ does the series $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n^p}$ converge?

**A:** all $p \leq 1$ **B:** all $p < 1$ **C:** all $p \geq 1$ **D:** all $p > 1$ **E:** all $p > 0$

5. The radius of convergence of the power series $\sum_{n=0}^{\infty} \dfrac{(-2)^n (x-5)^n}{n+1}$ is

**A:** $\infty$ **B:** $2$ **C:** $1$ **D:** $\dfrac{1}{2}$ **E:** $0$

6. Find $\lim_{x \to 0} \frac{\sin(x^2) - x^2}{x^6}$. [Hint: Use the Maclaurin series of $\sin(x^2)$.]

**A:** $\frac{1}{2}$ **B:** $-\frac{1}{2}$ **C:** $\frac{1}{6}$ **D:** $-\frac{1}{6}$ **E:** $0$

7. Find the coefficient of $(x-8)^2$ in the Taylor series of $f(x) = x^{\frac{4}{3}}$ about $x = 8$.

**A:** $-\frac{1}{9}$ **B:** $-\frac{1}{6}$ **C:** $\frac{1}{18}$ **D:** $0$ **E:** $\frac{2}{9}$

8. Assume the series $\sum_{n=0}^{\infty} c_n(x-3)^n$ converges for $x = 7$ and diverges for $x = 8$. Which of the following is true about the series?

**A:** The series converges at $x = -6$.
**B:** The series converges at $x = 0$.
**C:** The series diverges at $x = 2$.
**D:** The series diverges at $x = 0$.
**E:** We do not have enough information to decide since we don't know the $c_n$.

9. Let $f(x) = \frac{1}{2} e^{x^2}$. What is $f^{(2001)}(0)$?

**A:** $\frac{\frac{1}{2}}{(2001)!}$ **B:** $\frac{1}{2}(2001)!$ **C:** $\frac{(2001)!}{4000}$ **D:** $0$ **E:** $\frac{1}{2}(-1)^{2001}(2001)!$

10. Which of the following series represents $\frac{1}{x}$ in some appropriate interval?

**A:** $\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

**B:** $\sum_{n=0}^{\infty}(x+1)^n$

**C:** $\sum_{n=0}^{\infty}\frac{(x-1)^n}{2^{n+1}}$

**D:** $\sum_{n=0}^{\infty}(x-1)^n$

**E:** $\sum_{n=0}^{\infty}(-1)^n(x+1)^n$

**Part II: Work Out**
**Show all your work. Partial credit will be given.**
**You may use a calculator that cannot hold formulas.**

11. (20 points = 5 points for each part)
For each of the following series determine whether the series converges or diverges. Justify your answers fully by giving the appropriate test and showing how it applies to the series.

(a) $\sum_{n=1}^{\infty}(-1)^n\frac{n+1}{2n+1}$

(b) $\sum_{n=1}^{\infty}\frac{1}{2^n+n}$

11. (continued)

(c) $\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$

(d) $\sum_{n=1}^{\infty} \frac{\ln(n)}{2n}$

12. (10 points)
Consider the points $A(1,1,1)$, $B(0,2,0)$, $C(0,0,2)$.

(a) Show that the triangle $ABC$ is isosceles. (6 points)

(b) Compute the largest angle of the triangle $ABC$. (4 points)

13. (10 points)

Consider the series $\sum_{n=1}^{\infty} \frac{6}{(2n+1)(2n-1)}$.

(a) Compute the partial sums $s_1$, $s_2$, $s_3$. (3 points)

(b) Find a closed formula for $s_n$. [Hint: Use partial fractions to obtain a telescoping series.] (3 points)

(c) Use your answer to (b), or any other method, to decide whether the series converges or diverges. If it converges find its sum. (4 points)

14. (10 points = 5 points for each part)

(a) Find the Maclaurin series of $F(x) = \int_0^x \mathrm{e}^{-t^2}\mathrm{d}t$. (Write the general term.)

(b) Use this series to approximate $\int_0^{\frac{1}{4}} \mathrm{e}^{-t^2}\mathrm{d}t$ within $0.0005$. Justify your answer.