PART 1: MULTIPLE-CHOICE PROBLEMS

1. The area of the region bounded between the curves $y = 0$ and $y = \sin x$ from $x = \pi/4$ to $x = \pi/2$ is

$$\int_{\pi/4}^{\pi/2} \sin x \, dx = -\cos x \Big|_{\pi/4}^{\pi/2} = -\cos(\pi/2) + \cos(\pi/4) = -0 + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \, .$$

The correct answer is **(b)**.

2. The average value of the function $g(x) = \sqrt{1+2x}$ on the interval $[1, 4]$ is $\frac{1}{4-1}\int_1^4 \sqrt{1+2x}\, dx$. Using the substitution $u = 1 + 2x$ gives

$$\frac{1}{3}\int_1^4 \sqrt{1+2x}\, dx = \frac{1}{3}\int_3^9 \sqrt{u}\left(\frac{1}{2}\right) du = \frac{1}{6}\frac{u^{3/2}}{(3/2)}\Big|_3^9 = \frac{1}{9}\left(9^{3/2} - 3^{3/2}\right) = 3 - \frac{\sqrt{3}}{3} \, .$$

The correct answer is **(d)**.

3. Using the method of disks, the volume of the resulting ellipsoid is

$$V = \pi \int_{-2}^{2} y^2 \, dx = \pi \int_{-2}^{2} (36 - 9x^2) \, dx \, .$$

The correct answer is **(a)**.

4. Using the trigonometric substitution $x = 5\tan\theta$, then $dx = 5(\sec^2\theta)\, d\theta$ and

$$\int \frac{x^2}{\sqrt{x^2+25}}\, dx = \int \frac{25\tan^2\theta}{\sqrt{25\tan^2\theta + 25}} 5(\sec^2\theta)\, d\theta = \int \frac{25\tan^2\theta}{5\sec\theta} 5(\sec^2\theta)\, d\theta = 25\int (\tan^2\theta)(\sec\theta)\, d\theta \, .$$

The correct answer is **(a)**.

5. The integral $\int_0^4 \sqrt{16 - x^2}\, dx$ gives the area of a quarter circle of radius 4.

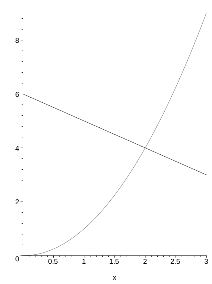
The correct answer is **(e)**.

6. Since $F(x)$ is an antiderivative of $F'(x)$, by the Fundamental Theorem of Calculus,

$$\int_0^3 F'(x)\,dx = F(x)\Big|_0^3 = F(3) - F(0) = 5 - 1 = 4$$

The correct answer is **(d)**.

7. The curves intersect at $x = 2$.



The area between the curves $y = x^2$ and $y = 6 - x$ from $x = 0$ to $x = 3$ is

$$\int_0^2 [(6-x) - x^2]\,dx + \int_2^3 [x^2 - (6-x)]\,dx = \int_0^2 (6 - x - x^2)\,dx + \int_2^3 (x^2 + x - 6)\,dx\ .$$

The correct answer is **(c)**.

8. A cross-section perpendicular to the y-axis and intersecting the y-axis at the point $(0, y)$ has area $A(y) = (2x)^2 = 4x^2 = 4[1 - (y^2/9)]$ The volume of the solid is

$$V = \int_{-3}^3 A(y)\,dy = \int_{-3}^3 4\left(1 - \frac{y^2}{9}\right)\,dy$$

The correct answer is **(b)**.

9. Let $u = x$ and $dv = e^{-x}\,dx$. Then $du = dx$ and $v = -e^{-x}$ and

$$\int_0^1 xe^{-x}\,dx = -xe^{-x}\Big|_0^1 - \int_0^1 -e^{-x}\,dx = -e^{-1} + \int_0^1 e^{-x}\,dx = -\frac{1}{e} - e^{-x}\Big|_0^1 = 1 - \frac{2}{e}\ .$$

The correct answer is **(e)**.

10. A circular cross-section through the point $(0, y)$ has area $A(y) = \pi\left(\sqrt{25-y^2}\right)^2 = \pi(25-y^2)$. The work in Joules required to empty the tank by pumping all of the water to the top of the tank is

$$\int_3^5 \rho g y\, A(y)\, dy = \rho g \int_3^5 \pi(25y - y^3)\, dy = \pi\rho g\left(\frac{25y^2}{2} - \frac{y^4}{4}\right)\Big|_3^5 = \pi\rho g\left[\frac{625}{4} - \frac{369}{4}\right] = 64\pi\rho g$$

The correct answer is **(c)**.

PART 2: WORK-OUT PROBLEMS

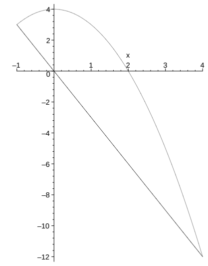
11. Let $u = (\ln x)^2$ and $dv = dx$. Then $du = 2(\ln x)(1/x)\, dx$ and $v = x$ and

$$\begin{aligned}\int (\ln x)^2\, dx &= x(\ln x)^2 - \int x[2(\ln x)(1/x)]\, dx \\ &= x(\ln x)^2 - 2\int \ln x\, dx \\ &= x(\ln x)^2 - 2(x\ln x - x) + C \\ &= x(\ln x)^2 - 2x\ln x + 2x + C\ .\end{aligned}$$

12. Let $x = 2\sec\theta$, then $dx = 2\sec\theta\tan\theta\, d\theta$ and

$$\begin{aligned}\int_2^{2\sqrt{2}} \frac{\sqrt{x^2-4}}{x}\, dx &= \int_0^{\pi/4} \frac{\sqrt{4\sec^2\theta - 4}}{2\sec\theta} 2\sec\theta\tan\theta\, d\theta \\ &= 2\int_0^{\pi/4} \tan^2\theta\, d\theta \\ &= 2\int_0^{\pi/4} (\sec^2\theta - 1)\, d\theta \\ &= 2(\tan\theta - \theta)\Big|_0^{\pi/4} = 2 - \frac{\pi}{2}\end{aligned}$$

13. The region R bounded by the curves $y = 4 - x^2$ and $y = -3x$ is shown below.



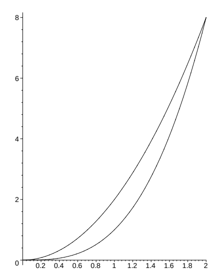
(a) An integral with respect to x that gives the area of the region R is

$$\int_{-1}^{4} [(4 - x^2) - (-3x)]\, dx = \int_{-1}^{4} (4 + 3x - x^2)\, dx = \left(4x + \frac{3x^2}{2} - \frac{x^3}{3}\right)\Big|_{-1}^{4} = \frac{56}{3} - \left(-\frac{13}{6}\right) = \frac{125}{6} .$$

(b) An expression involving integration with respect to y that represents the area of the region R is

$$\int_{-12}^{3} [\sqrt{4-y} - (-y/3)]\, dy + \int_{3}^{4} [\sqrt{4-y} - (-\sqrt{4-y})]\, dy = \int_{-12}^{3} [\sqrt{4-y} + (y/3)]\, dy + \int_{3}^{4} 2\sqrt{4-y}\, dy .$$

14. The region R bounded between the curves $y = x^3$ and $y = 2x^2$ is shown below.



(a) The volume of the solid obtained by revolving the region R about the x-axis is

$$\pi \int_{0}^{2} [(2x^2)^2 - (x^3)^2]\, dx = \pi \int_{0}^{2} [4x^4 - x^6]\, dx = \pi \left(\frac{4x^5}{5} - \frac{x^7}{7}\right)\Big|_{0}^{2} = \frac{256\pi}{35} .$$

(b) The volume of the solid obtained by revolving the region R about the y-axis is

$$2\pi \int_{0}^{2} x[2x^2 - x^3]\, dx = 2\pi \int_{0}^{2} [2x^3 - x^4]\, dx = 2\pi \left(\frac{x^4}{2} - \frac{x^5}{5}\right)\Big|_{0}^{2} = \frac{16\pi}{5} .$$

15. Let $u = \sec x$, then $du = \sec x \, \tan x \, dx$ and

$$\begin{aligned}
\int (\sec^5 x)(\tan^3 x)\, dx &= \int (\sec^4 x)(\tan^2 x) \sec x \, \tan x \, dx \\
&= \int (\sec^4 x)(\sec^2 x - 1) \sec x \, \tan x \, dx \\
&= \int u^4 (u^2 - 1)\, du \\
&= \int (u^6 - u^4)\, du \\
&= \frac{u^7}{7} - \frac{u^5}{5} + C \\
&= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C \; .
\end{aligned}$$