VERSION A SOLUTIONS

PART 1: MULTIPLE-CHOICE PROBLEMS

1. The differential equation is separable.

$$\begin{aligned}
\frac{dy}{dx} &= xy \\
\frac{1}{y}\,dy &= x\,dx \\
\int \frac{1}{y}\,dy &= \int x\,dx + C \\
\ln y &= \frac{x^2}{2} + C \\
y &= e^{(x^2/2)+C} \ .
\end{aligned}$$

Now $y(0) = 2$, so $C = \ln 2$ and we have that $y(x) = 2e^{(x^2/2)}$. Thus, $y(2) = 2e^2$ and the correct answer is **(c)**.

2. Now

$$\frac{2}{x(x+2)} = \frac{1}{x} - \frac{1}{x+2}$$

$$\int \frac{2}{x(x+2)}\,dx = \int \frac{1}{x}\,dx - \int \frac{1}{x+2}\,dx = \ln|x| - \ln|x+2| + C \ .$$

The correct answer is **(b)**.

3. The area of the surface generated by rotating the curve $y = x^2$ from $x = 0$ to $x = \sqrt{2}$ about the y-axis is

$$\int_0^{\sqrt{2}} 2\pi x\sqrt{1+(dy/dx)^2}\,dx = \int_0^{\sqrt{2}} 2\pi x\sqrt{1+(2x)^2}\,dx = \int_0^{\sqrt{2}} 2\pi x\sqrt{1+4x^2}\,dx \ .$$

The correct answer is **(a)**.

4. Now

$$0 \le \frac{\cos^2 x}{x^2+\sqrt{x}} \le \frac{1}{x^2}$$

for all $x \ge 1$ and $\displaystyle\int_1^{\infty} \frac{1}{x^2}\,dx$ converges. The correct answer is **(a)**.

5. The center of mass is located at

$$\bar{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3} = \frac{(2)(-2)+(4)(1)+(6)(4)}{2+4+6} = \frac{24}{12} = 2 \ .$$

The correct answer is **(e)**.

6. The partition of $[1,13]$ when $n=3$ is $\{1,5,9,13\}$. So $\Delta x = 4$ and the midpoints of the subintervals are $3,7$ and 11. Thus, $M_3 = (4)[(1/3)+(1/7)+(1/11)]$.

   The correct answer is **(d)**.

7. The length of the curve $y=(2/3)x^{3/2}$ from $x=0$ to $x=8$ is

$$\int_0^8 \sqrt{1+(dy/dx)^2}\,dx = \int_0^8 \sqrt{1+(\sqrt{x})^2}\,dx = \int_0^8 \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}\Big|_0^8 = \frac{52}{3}\ .$$

   The correct answer is **(c)**.

8. The length of the curve $x=2t+t^2$, $y=2t-t^2$ for $0 \le t \le 3$ is

$$\int_0^3 \sqrt{(dx/dt)^2+(dy/dt)^2}\,dt = \int_0^3 \sqrt{(2+2t)^2+(2-2t)^2}\,dt = \int_0^3 \sqrt{8(1+t^2)}\,dt\ .$$

   The correct answer is **(d)**.

9. The hydrostatic force (in Newtons) on one side of the vertical rectangular plate is

$$\rho g \int_9^{12} 10y\,dy = 9800 \int_9^{12} 10y\,dy = 9800 \times 315\ .$$

   The correct answer is **(b)**.

10. If $y(t)$ is the amount of salt (in kilograms) in the tank after t minutes, then

$$\frac{dy}{dt} = \text{rate in} - \text{rate out} = 0 - \left(\frac{y(t)}{1000}\frac{\text{kg}}{\text{L}}\right)\left(20\frac{\text{L}}{\text{min}}\right) = -\frac{y(t)}{50}\frac{\text{kg}}{\text{min}}\ .$$

   Now $y(0)=10$ kg and the correct answer is **(e)**.

PART 2: WORK-OUT PROBLEMS

11. Now

$$\frac{x+16}{x^3+4x} = \frac{x+16}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$
$$= \frac{Ax^2+4A+Bx^2+Cx}{x^3+4x}$$
$$= \frac{(A+B)x^2+Cx+4A}{x^3+4x} ,$$

so

$$A+B=0$$
$$C=1$$
$$4A=16$$

the solution of which is $A=4$, $B=-4$ and $C=1$. Thus

$$\int \frac{x+16}{x^3+4x}\,dx = \int \left(\frac{4}{x} + \frac{1-4x}{x^2+4}\right)\,dx$$
$$= \int \frac{4}{x}\,dx + \int \frac{1}{x^2+4}\,dx - \int \frac{4x}{x^2+4}\,dx$$
$$= 4\ln|x| + \frac{1}{2}\tan^{-1}(x/2) - 2\ln(x^2+4) + C .$$

12. The differential equation $\frac{dy}{dx} - (2/x)y = x$ has integrating factor $I(x) = e^{\int -(2/x)\,dx} = e^{-2\ln x} = x^{-2}$. Thus,

$$x^{-2}\left(\frac{dy}{dx} - (2/x)y\right) = x^{-1}$$
$$\frac{d}{dx}(x^{-2}y) = x^{-1}$$
$$x^{-2}y = \ln x + C$$
$$y = x^2(\ln x + C) .$$

Now $2 = y(1) = (1)^2(\ln 1 + C) = C$, so $y = x^2(2+\ln x)$.

13. An integration by parts gives

$$\int_1^b xe^{-x}\,dx = -xe^{-x}\Big|_1^b + \int_1^b e^{-x}\,dx = 2e^{-1} - be^{-b} - e^{-b}\ .$$

Now $\lim_{b\to\infty} be^{-b} = 0$ and $\lim_{b\to\infty} e^{-b} = 0$, so

$$\int_1^\infty xe^{-x}\,dx = \lim_{b\to\infty}\int_1^b xe^{-x}\,dx = \lim_{b\to\infty}(2e^{-1} - be^{-b} - e^{-b}) = 2e^{-1}\ .$$

14. The x-coordinate of the centroid of the region in the first quadrant that is bounded by the curves $y = 4 - x^2$, $y = 0$ and $x = 0$ is

$$\bar{x} = \frac{\displaystyle\int_0^2 x(4-x^2)\,dx}{\displaystyle\int_0^2 4 - x^2\,dx} = \frac{\displaystyle\int_0^2 4x - x^3\,dx}{\displaystyle\int_0^2 4 - x^2\,dx} = \frac{\left(2x^2 - \dfrac{x^4}{4}\right)\Big|_0^2}{\left(4x - \dfrac{x^3}{3}\right)\Big|_0^2} = \frac{3}{4}\ .$$

15. The partition of $[1,4]$ when $n = 6$ is $\{1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0\}$. So $\Delta x = 1/2$ and

$$\begin{aligned}\int_1^4 f(x)\,dx \approx S_6 &= \frac{\Delta x}{3}[f(1.0) + 4f(1.5) + 2f(2.0) + 4f(2.5) + 2f(3.0) + 4f(3.5) + f(4.0)]\\ &= \frac{1}{6}[(1/2) + 4(1/4) + 2(1/8) + 4(1/6) + 2(1/6) + 4(1/2) + (1/4)]\\ &= \frac{5}{6}\ .\end{aligned}$$

Now $|f^{(4)}(x)| \le 60$ for all $1 \le x \le 4$, so

$$|E_S| \le \frac{60(4-1)^5}{180(6)^4} = \frac{1}{16}\ .$$