PART 1: MULTIPLE-CHOICE PROBLEMS

Each problem is worth 5 points: NO partial credit will be given. The use of calculators is prohibited.

1. Objects with masses $m_1 = 2$, $m_2 = 4$ and $m_3 = 6$ are located along the x-axis at the points $x_1 = -2$, $x_2 = 1$ and $x_3 = 4$ respectively. The center of mass is located at $\bar{x} =$

   (a) $\frac{3}{8}$

   (b) $\frac{8}{3}$

   (c) 2

   (d) $\frac{1}{2}$

   (e) 0

2. Which integral gives the area of the surface generated by rotating the curve $y = x^2$ from $x = 0$ to $x = \sqrt{2}$ about the y-axis?

   (a) $\int_0^2 \pi\sqrt{4y^2 + y}\, dy$

   (b) $\int_0^{\sqrt{2}} \pi\sqrt{4y + 1}\, dy$

   (c) $\int_0^{\sqrt{2}} 2\pi x\sqrt{1 + x^2}\, dx$

   (d) $\int_0^{\sqrt{2}} 2\pi x^2\sqrt{1 + 4x^2}\, dx$

   (e) $\int_0^{\sqrt{2}} 2\pi x\sqrt{1 + 4x^2}\, dx$

3. Find $\int \frac{2}{x(x+2)}\, dx$

   (a) $\ln|x| + \ln|x+2| + C$

   (b) $\ln|x+2| - \ln|x| + C$

   (c) $\ln|x| - \ln|x+2| + C$

   (d) $\ln|x+1| - \ln|x+2| + C$

   (e) $\ln|x+1| + \ln|x| + C$

4. The approximation to $\int_1^{13} \frac{1}{x}\, dx$ obtained by using the Midpoint Rule with $n = 3$ is

(a) $4\left(1 + \frac{1}{5} + \frac{1}{9}\right)$

(b) $4\left(\frac{1}{3} + \frac{1}{7} + \frac{1}{11}\right)$

(c) $4\left(\frac{1}{5} + \frac{1}{9} + \frac{1}{13}\right)$

(d) $\frac{13}{3}\left(\frac{1}{3} + \frac{1}{7} + \frac{1}{11}\right)$

(e) $2\left(\frac{1}{3} + \frac{1}{7} + \frac{1}{11}\right)$

5. Which integral gives the length of the curve $x = 2t + t^2$, $y = 2t - t^2$ for $0 \le t \le 3$?

(a) $\int_0^3 \sqrt{8(1+t^2)}\, dt$

(b) $\int_0^3 \sqrt{8}(1+t)\, dt$

(c) $\int_0^3 \sqrt{8(1+2t^2)}\, dt$

(d) $\int_0^3 2\sqrt{1+t^2}\, dt$

(e) $\int_0^3 2(1+t)\, dt$

6. The improper integral $\int_1^{\infty} \frac{\cos^2 x}{x^2 + \sqrt{x}}\, dx$

(a) Converges by comparison to $\int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$

(b) Converges by comparison to $\int_1^{\infty} \frac{1}{x^2}\, dx$

(c) Diverges by comparison to $\int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$

(d) Diverges by comparison to $\int_1^{\infty} \frac{1}{x^2}\, dx$

(e) Converges to 1

7. Find the length of the curve $y = (2/3)x^{3/2}$ from $x = 0$ to $x = 8$.

(a) 18

(b) $\dfrac{26}{3}$

(c) $\dfrac{56}{3}$

(d) $\dfrac{52}{3}$

(e) 26

8. If $\dfrac{dy}{dx} = xy$ and $y(0) = 2$, then $y(2) =$

(a) $2e^2$

(b) e^2

(c) $2e^4$

(d) $\dfrac{e^4}{2}$

(e) e^4

9. A tank contains 10 kg of salt dissolved in 1000 L of water. Pure water enters the tank at a rate of 20 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. Let $y(t)$ be the amount of salt (in kilograms) in the tank after t minutes. The initial value problem satisfied by $y(t)$ is

(a) $\dfrac{dy}{dt} = -\dfrac{y}{100}\,, \quad y(0) = 20$

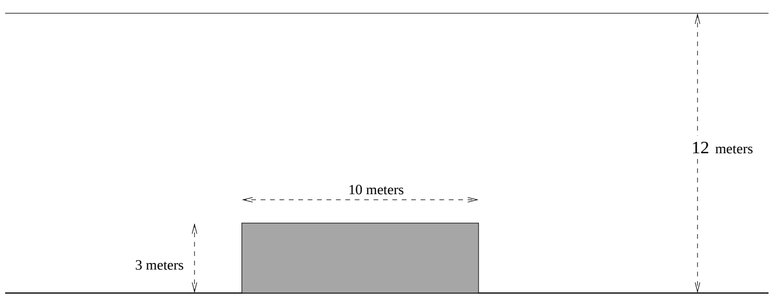
(b) $\dfrac{dy}{dt} = \dfrac{1}{100} - \dfrac{y}{50}\,, \quad y(0) = 10$

(c) $\dfrac{dy}{dt} = \dfrac{1}{100} + \dfrac{y}{50}\,, \quad y(0) = 10$

(d) $\dfrac{dy}{dt} = -\dfrac{y}{50}\,, \quad y(0) = 10$

(e) $\dfrac{dy}{dt} = \dfrac{y}{50}\,, \quad y(0) = 10$

10. Find the hydrostatic force (in Newtons) on one side of the vertical rectangular plate shown below standing at the bottom of a pool of water that is 12 meters deep. The acceleration due to gravity is $g = 9.8$ m/sec^2 and the density of water is $\rho = 1000$ kg/m^3.



(a) 9800×62.5

(b) 9800×10

(c) 9800×100

(d) 9800×31.5

(e) 9800×315

PART 2: WORK-OUT PROBLEMS

Each problem is worth 10 points; partial credit is possible. The use of calculators is prohibited. **SHOW ALL WORK!**

11. Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2y}{x} + x$$
$$y(1) = 2\ .$$

12. Use *partial fractions* to evaluate $\displaystyle\int \frac{x+18}{x^3+9x}\,dx$

13. Find the x-coordinate of the centroid of the region in the first quadrant that is bounded by the curves $y = 4 - x^2$, $y = 0$ and $x = 0$.

14. Evaluate the integral $\int_1^\infty xe^{-x}\,dx$. You must clearly justify all conclusions in order to receive full credit.

15. Suppose that the following data for the function $y = f(x)$ were obtained from an experiment:

x	1	4/3	5/3	2	7/3	8/3	3
y	1/2	1/8	1/2	1/6	1/6	1/4	1/2

(a) Use Simpson's Rule with $n = 6$ to approximate $\int_1^3 f(x)\,dx$.

(6 points)

(b) Given that $20 \leq f^{(4)}(x) \leq 80$ for all $1 \leq x \leq 3$, find the maximum possible error that results by using Simpson's Rule in part (a).

(4 points)