1. Find the average value of \( f(x) = \cos x \) on the interval \( -\frac{\pi}{4} \leq x \leq \frac{\pi}{4} \).
   a. \( \frac{2\sqrt{2}}{\pi} \)
   b. \( \sqrt{2} \frac{\pi}{4} \)
   c. \( \sqrt{2} \)
   d. \( \frac{1}{\sqrt{2}} \)
   e. \( \pi \frac{\sqrt{2}}{\sqrt{2}} \)

2. The ellipse \( \frac{x^2}{4} + \frac{y^2}{16} = 1 \) is revolved about the \( x \)-axis. Which integral gives the volume of the resulting ellipsoid?
   a. \( \int_{-2}^{2} 2\pi x \sqrt{16 - 4x^2} \, dx \)
   b. \( \int_{-4}^{4} 2\pi (16 - 4x^2)^2 \, dx \)
   c. \( \int_{-2}^{2} \pi (16 - 4x^2) \, dx \)
   d. \( \int_{-4}^{4} 2\pi x \sqrt{16 - 4x^2} \, dx \)
   e. \( \int_{-2}^{2} \pi (16 - 4x^2)^2 \, dx \)

3. Compute \( \int_{0}^{\pi/4} \cos \theta \sin^3 \theta \, d\theta \).
   a. \( \frac{1}{2} \)
   b. \( \frac{1}{4} \)
   c. \( \frac{1}{8} \)
   d. \( \frac{1}{16} \)
   e. \( \frac{1}{32} \)
4. Compute \( \int_{0}^{\ln 2} xe^{-x} \, dx \).
   
   a. \( \frac{1}{2} + \frac{1}{2} \ln 2 \)
   b. \( \frac{1}{2} - \frac{1}{2} \ln 2 \)
   c. \( \frac{1}{2} \ln 2 - \frac{1}{2} \)
   d. \( -\frac{1}{2} \ln 2 - \frac{1}{2} \)
   e. Divergent

5. Use the Trapezoid Rule with \( n = 4 \) intervals to approximate the integral \( \int_{1}^{9} (9 + x^2) \, dx \).
   
   a. 240
   b. 312
   c. 314 \( \frac{1}{3} \)
   d. 320
   e. 400

6. A barrel initially contains 3 cups of sugar dissolved in 4 gallons of water. You then add pure water at the rate of 2 gallons per minute while the mixture is draining out of a hole in the bottom at 2 gallons per minute. Find the amount of sugar in the barrel after 2 minute.
   
   a. \( \frac{3}{\sqrt{e}} \)
   b. \( \frac{3}{e} \)
   c. \( 3e \)
   d. \( 3 \sqrt{e} \)
   e. \( \frac{3}{e^2} \)
7. As \( n \) approaches infinity, the sequence \( \left\{ \frac{1 - \cos n}{n^2} \right\} \)

   a. converges to \(-\frac{1}{2}\)
   b. converges to 0
   c. converges to \(\frac{1}{2}\)
   d. converges to 1
   e. diverges

8. Compute \( \sum_{n=1}^{\infty} \left( \frac{n}{n+1} - \frac{n+1}{n+2} \right) \)

   a. \(-\frac{1}{2}\)
   b. \(\frac{1}{2}\)
   c. 1
   d. 2
   e. Divergent

9. Find the 4th degree Taylor polynomial for \( f(x) = x^2 - x \) about \( x = 2 \).

   a. \( T_4(x) = 2 + 3(x - 2) + (x - 2)^2 + 3(x - 2)^3 + (x - 2)^4 \)
   b. \( T_4(x) = 2 + 3(x - 2) + 2(x - 2)^2 + 3(x - 2)^3 + 2(x - 2)^4 \)
   c. \( T_4(x) = 2 + 3(x - 2) + (x - 2)^2 \)
   d. \( T_4(x) = 2 + 3(x - 2) + 2(x - 2)^2 \)
   e. \( T_4(x) \) cannot be found because \( x = 2 \) is outside the interval of convergence.
10. A triangle has vertices $A = (0, 3, 2)$, $B = (-2, 3, 0)$ and $C = (-2, 0, 3)$. Find the angle at vertex $B$.

   a. $\frac{\pi}{6}$
   b. $\frac{\pi}{3}$
   c. $\frac{\pi}{2}$
   d. $\frac{2\pi}{3}$
   e. $\frac{5\pi}{6}$

11. If $\vec{u}$ points South-West and $\vec{v}$ points Up, which way does $\vec{u} \times \vec{v}$ point?

   a. South-East
   b. North-East
   c. North-West
   d. $45^\circ$ Up from North-West
   e. $45^\circ$ Down from North-West

12. Find the area of a triangle with edges $\vec{a} = (3, -2, 1)$ and $\vec{b} = (-1, 0, 1)$.

   a. 1
   b. 2
   c. $\sqrt{6}$
   d. 6
   e. $2\sqrt{6}$
13. (12 points) The end of a water trough occupies
the region between \( y = x^2 \) m and \( y = 9 \) m.
It is filled to a depth of \( y = 4 \) m.
Find the force on the end of the trough.
Give your answer in terms of \( \rho \) (the density of water)
and \( g \) (the acceleration of gravity).

14. (12 points) Compute \( \int_{3}^{3x} \frac{\sqrt{x^2 - 9}}{x} \, dx. \)
15. (12 points) Find the arc length of the curve \( y = \frac{x^2}{4} - \frac{\ln x}{2} \) between \( x = 1 \) and \( x = e \).

16. (12 points) The Taylor series \( f(x) = \sum_{n=1}^{\infty} \frac{H_n}{2^n} (x - 1)^{n-1} \) is obtained by differentiating the series \( g(x) = \sum_{n=0}^{\infty} \frac{(x - 1)^n}{2^n} = \sum_{n=0}^{\infty} \left( \frac{x - 1}{2} \right)^n \). What is the function \( f(x) \)? What is the interval of convergence for \( f(x) \) (including endpoints)? Justify your answers.