MATH 152, SPRING SEMESTER 2005
COMMON EXAMINATION II - VERSION A

Name (print):______________________________

Signature: ______________________________

Instructor's name: ______________________________

Section No: ______________________________

Seat No: ______________________________

INSTRUCTIONS

1. Calculators may _not_ be used on this exam. The ScanTrons will be collected after 90 minutes.
2. In Part 1 (Problems 1–10), mark the correct choice on your ScanTron form using a No.2 pencil. _For your own record, mark your choices on the exam itself_, as the ScanTrons will _not_ be returned.
3. In Part 2 (Problems 11–15), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

Possibly useful formulas:

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}, \text{ where } K = \max|f''(x)| \text{ for } a \leq x \leq b$$

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}, \text{ where } K = \max|f''(x)| \text{ for } a \leq x \leq b$$

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}, \text{ where } K = \max|f^{(4)}(x)| \text{ for } a \leq x \leq b$$

QN	PTS
1–10	
11	
12	
13	
14	
15	
TOTAL	

Part 1 – Multiple Choice (50 points)

Read each question carefully; each problem is worth **5 points**.

1. Find the x-coordinate of the center of mass of the system of objects that have a mass of 3, 5, and 7 at the points $(1, 2)$, $(-2, 5)$ and $(3, 1)$, respectively.

(a) $\frac{38}{15}$

(b) 14

(c) $\frac{15}{14}$

(d) $\frac{14}{15}$

(e) 0

2. An integrating factor for the differential equation $(x^2+1)\frac{dy}{dx} + 4xy = x^3$ is:

(a) $I(x) = (x^2+1)^2$

(b) $I(x) = e^{2x^2}$

(c) $I(x) = \ln(x^2+1)$

(d) $I(x) = 2x^2 + 2$

(e) $I(x) = 4x \arctan x$

3. The length of the curve $x = 3t + 1$, $y = 4 - t$, $1 \le t \le 3$ is:

(a) $10\sqrt{3}$

(b) $4\sqrt{10}$

(c) $\frac{2}{3}(31\sqrt{31} - 11\sqrt{11})$

(d) $\frac{1}{15}(31\sqrt{31} - 11\sqrt{11})$

(e) $2\sqrt{10}$

4. If the Trapezoid Rule with $n = 4$ is used to approximate $\int_2^5 \ln x \, dx$, then the Trapezoid error formula says the error in the approximation is bounded by

(a) $|E_T| < \dfrac{1}{64}$

(b) $|E_T| < \dfrac{9}{256}$

(c) $|E_T| < \dfrac{1}{256}$

(d) $|E_T| < \dfrac{9}{64}$

(e) $|E_T| < \dfrac{1}{1024}$

5. Which of the following integrals gives the surface area obtained by rotating the curve $y = \ln(4x)$, $1 \leq x \leq 3$ about the y-axis?

(a) $\displaystyle\int_1^3 2\pi x\sqrt{1+\frac{1}{x^2}}\,dx$

(b) $\displaystyle\int_1^3 2\pi \ln(4x)\sqrt{1+\frac{1}{x^2}}\,dx$

(c) $\displaystyle\int_1^3 2\pi x\sqrt{1+\frac{1}{16x^2}}\,dx$

(d) $\displaystyle\int_1^3 2\pi \ln(4x)\sqrt{1+\frac{1}{16x^2}}\,dx$

(e) $\displaystyle\int_1^3 2\pi x\sqrt{1+\ln^2 4x}\,dx$

6. The improper integral $\displaystyle\int_1^\infty \frac{1}{x^2+1}\,dx$

(a) Converges to 0.

(b) Converges to 1.

(c) Converges to $\dfrac{\pi}{4}$.

(d) Converges to $\dfrac{\pi}{2}$.

(e) Diverges.

7. If a curve passes thru the point $(0, \frac{1}{2})$ and has the property that the slope of the curve at each point (x, y) is x^3y^2, find the equation of the curve.

(a) $\frac{4}{8-x^4}$

(b) $\frac{4}{8-x^2}$

(c) $\frac{4}{x^4-8}$

(d) $\frac{1}{2} - \frac{x^4}{4}$

(e) $\frac{x^2}{4} + \frac{1}{2}$

8 Consider the integral $\int_0^\infty \frac{1}{\sqrt{x}+e^{10x}}\,dx$. Which of the following is true?

(a) There is insufficient information to make a conclusion.

(b) The integral diverges to infinity.

(c) The integral converges to 0.

(d) The integral converges to a value $L \leq \frac{1}{10}$

(e) The integral converges to a value $L > \frac{1}{10}$

9. Find the length of the curve $y = \frac{2}{3}(x-1)^{\frac{3}{2}}$, $1 \leq x \leq 4$.

(a) $2\sqrt{3}$

(b) 9

(c) $\frac{12}{5}\sqrt{3}$

(d) $\frac{14}{3}$

(e) $\sqrt{3}$

10. Use The Trapezoid Rule rule with $n = 4$ to approximate $\int_1^2 \ln x \, dx$.

(a) $\frac{1}{8}(\ln \frac{5}{4} + 2\ln \frac{3}{2} + 2\ln \frac{7}{4} + \ln 2)$

(b) $\frac{1}{8}(2\ln \frac{5}{4} + 2\ln \frac{3}{2} + 2\ln \frac{7}{4} + \ln 2)$

(c) $\frac{1}{12}(4\ln \frac{5}{4} + 2\ln \frac{3}{2} + 4\ln \frac{7}{4} + \ln 2)$

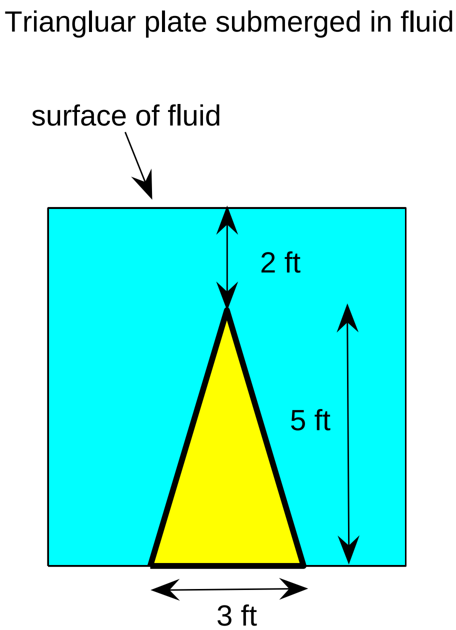
(d) $\frac{1}{4}(\ln \frac{5}{4} + 2\ln \frac{3}{2} + 2\ln \frac{7}{4} + \ln 2)$

(e) $\frac{1}{8}(2\ln \frac{5}{4} + 2\ln \frac{3}{2} + 2\ln \frac{7}{4} + 2\ln 2)$

Part 2 (56 points)

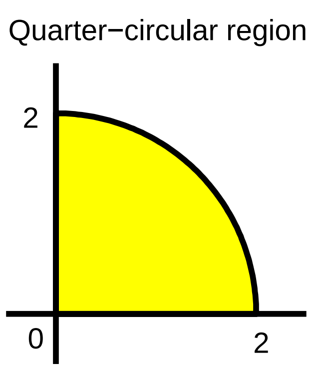
The use of a calculator is NOT permitted for this part of the exam. <u>All work must be shown</u> in order to receive credit. Refer to the front page for further instructions.

11. (`12 points`) A triangular shaped plate is submerged vertically in a fluid with weight density $\delta = \rho g = 60$ pounds per cubic feet as shown. Find the force of the fluid on one side of the plate.



12. (`10 points`) Find the surface area obtained by rotating the curve $x = y^2 + 1$, $1 \le y \le 2$ about the x-axis.

13. (`10 points`) Find the x-coordinate of the centroid of the quarter-circular region depicted.



14. (`14 points`) Solve the differential equations. Solve explicitly for y.

a.) $\dfrac{dy}{dt} = 1 + t - yt - y$

b.) $\dfrac{dy}{dx} - 2y = x$, $y(0) = 1$.

15. (`10 points`) A tank contains 1000 liters of pure water. Salt water that contains 0.5 kg of salt per liter of water enters the tank at a rate of 10 liters per minute. The solution is kept mixed and exits the tank at the same rate. How much salt is in the tank after 10 minutes?