

Name _____ Sec _____ ID _____

MATH 152

Final Exam

Fall 2007

Sections 513 - 515

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Multiple Choice: (6 points each)

1-11	/66
12	/12
13	/15
14	/10
Total	/103

1. A plate is bounded by the curves $y = x^2$, $y = -x^2$ and $x = 2$ measured in meters. Find the total mass of the plate if the surface density is $\rho = 3 \text{ kg/m}^2$.

- a. $\frac{8}{3}$
- b. $\frac{16}{3}$
- c. $\frac{32}{3}$
- d. 8
- e. 16

2. A plate is bounded by the curves $y = x^2$, $y = -x^2$ and $x = 2$ measured in meters. Find the center of mass of the plate if the surface density is $\rho = 3 \text{ kg/m}^2$.

- a. $\left(\frac{3}{4}, 0\right)$
- b. $\left(\frac{3}{4}, \frac{1}{10}\right)$
- c. $\left(\frac{3}{4}, \frac{6}{5}\right)$
- d. $\left(\frac{3}{2}, 0\right)$
- e. $\left(\frac{3}{2}, \frac{6}{5}\right)$

3. Compute $\int x^2 e^{2x} dx$.

a. $\frac{x^2}{2} e^{2x} - \frac{x}{2} e^{2x} - \frac{1}{4} e^{2x} + C$

b. $\frac{x^2}{2} e^{2x} - \frac{x}{2} e^{2x} + \frac{1}{4} e^{2x} + C$

c. $\frac{x^2}{2} e^{2x} - \frac{x}{2} e^{2x} - \frac{1}{2} e^{2x} + C$

d. $\frac{x^2}{2} e^{2x} - \frac{x}{2} e^{2x} + \frac{1}{2} e^{2x} + C$

e. $\frac{x^2}{2} e^{2x} - \frac{x}{4} e^{2x} - \frac{1}{4} e^{2x} + C$

4. The parametric curve $x = 2t^2$ $y = t^3$ for $0 \leq t \leq 1$ is rotated about the y -axis. Which integral gives the area of the surface swept out?

a. $2\pi \int_0^1 t \sqrt{16 + 9t^2} dt$

b. $2\pi \int_0^1 t^3 \sqrt{16 + 9t^2} dt$

c. $4\pi \int_0^1 t^3 \sqrt{16 + 9t^2} dt$

d. $2\pi \int_0^1 t^4 \sqrt{16 + 9t^2} dt$

e. $4\pi \int_0^1 t^4 \sqrt{16 + 9t^2} dt$

5. Which term appears in the partial fraction expansion of $\frac{3x^2 - 4x - 20}{(x - 2)^2(x^2 + 4)}$?

a. $\frac{-2}{(x - 2)^2}$

b. $\frac{1}{(x - 2)^2}$

c. $\frac{2}{(x - 2)^2}$

d. $\frac{-2}{(x - 2)}$

e. $\frac{1}{(x - 2)}$

6. Find the solution of the differential equation $x \frac{dy}{dx} = 2y + x^3$ satisfying the initial condition $y(1) = 2$.

a. $y = x^3 - \frac{7}{4}x^2$

b. $y = x^3 + x^2$

c. $y = 2x^3 - x^2$

d. $y = \frac{9}{5x^2} + \frac{1}{5}x^3$

e. $y = \frac{5}{3x^2} + \frac{1}{3}x^3$

7. The region in the first quadrant bounded by the curves $y = x^2$, $y = 0$ and $x = 2$ is rotated about the y -axis. Find the volume of the solid swept out.

- a. $\frac{32}{5}\pi$
- b. $\frac{64}{5}\pi$
- c. 8π
- d. 4π
- e. 2π

8. If you approximate $f(x) = \ln(x)$ on the interval $\left[\frac{1}{2}, \frac{3}{2}\right]$ by its 3rd degree Taylor polynomial centered at $x = 1$, namely $T_3(x) = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3$, then the Taylor Remainder Theorem says the error $|R_3(x)|$ is less than

Taylor Remainder Theorem:

If $T_n(x)$ is the n^{th} degree Taylor polynomial for $f(x)$ centered at $x = a$ then there is a number c between a and x such that the remainder is

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{(n+1)}$$

- a. 1
- b. $\frac{1}{2}$
- c. $\frac{1}{4}$
- d. $\frac{1}{8}$
- e. 0

9. Compute $\lim_{x \rightarrow 0} \frac{\sin(x^2) - x^2}{e^{(x^3)} - 1 - x^3}$.

a. $-\frac{1}{6}$

b. $\frac{1}{6}$

c. $\frac{2}{3}$

d. $-\frac{1}{3}$

e. $\frac{1}{3}$

10. Find the volume of the parallelepiped with edge vectors

$$\vec{a} = (-2, 2, 1), \quad \vec{b} = (3, 2, 4) \quad \text{and} \quad \vec{c} = (1, -2, 3)$$

a. -26

b. 26

c. $\sqrt{26}$

d. -46

e. 46

11. If \vec{u} points Down and \vec{v} points North West, in which direction does $\vec{u} \times \vec{v}$ point?

a. North East

b. South

c. South West

d. South East

e. Up

Work Out: (Points indicated. Part credit possible.)

12. (12 points) Compute $\int_0^1 \frac{x^2}{\sqrt{4-x^2}} dx$

13. (15 points) A cone of radius 4 and height 6 and vertex down is filled with water up to height 2. Find the work done to pump the water out the top. Give your answer as a multiple of ρg .

14. (10 points) Start from the Maclaurin series: $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$

a. (6 pt) Find the Maclaurin series for $\frac{1}{(1+x)^2}$.

HINT: Differentiate both sides of the given series.

b. (4 pt) Compute $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2^{n-1}}$.