MATH 152 Spring 2016
COMMON EXAM I - VERSION A

LAST NAME: Key FIRST NAME: ______________________

INSTRUCTOR: ______________________

SECTION NUMBER: ____________

UIN: ______________________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 4 points.
4. In Part 2 (Problems 16-20), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		8
17		8
18		9
19		9
20		21
Total		100

PART I: Multiple Choice. 3 points each.

1. Find the area bounded by $y = e^x$, $y = e^{-x}$, $x = 0$ and $x = 1$.

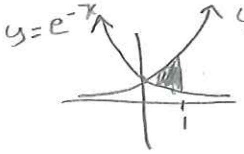
(a) $1 + \frac{1}{e}$

(b) $e + \frac{1}{e} + 2$

(c) $1 + \frac{1}{e} - 2$

(d) $e + \frac{1}{e} - 2$

(e) $e - \frac{1}{e}$



$$A = \int_0^1 (e^x - e^{-x})\,dx$$
$$= (e^x + e^{-x})\Big|_0^1$$
$$= e + e^{-1} - (1+1)$$
$$= e + \frac{1}{e} - 2$$

2. Find the average value of $f(x) = \sin^2 x$ on the interval $\left[0, \frac{\pi}{8}\right]$.

(a) $\frac{1}{2}\left(\frac{\pi}{8} - \frac{\sqrt{2}}{4}\right)$

(b) $\frac{4}{\pi}\left(\frac{\pi}{8} + \frac{\sqrt{2}}{4}\right)$

(c) $\frac{1}{2}\left(\frac{\pi}{8} + \frac{\sqrt{2}}{4}\right)$

(d) $\frac{4}{\pi}\left(\frac{\pi}{8} - \sqrt{2}\right)$

(e) $\frac{4}{\pi}\left(\frac{\pi}{8} - \frac{\sqrt{2}}{4}\right)$

$$f_{ave} = \frac{8}{\pi}\int_0^{\frac{\pi}{8}} \sin^2 x\,dx$$
$$= \frac{8}{\pi}\cdot\frac{1}{2}\int_0^{\frac{\pi}{8}} (1 - \cos 2x)\,dx$$
$$= \frac{4}{\pi}\left(x - \frac{1}{2}\sin 2x\right)\Big|_0^{\frac{\pi}{8}}$$
$$= \frac{4}{\pi}\left(\frac{\pi}{8} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - 0\right)$$
$$= \frac{4}{\pi}\left(\frac{\pi}{8} - \frac{\sqrt{2}}{4}\right)$$

3. $\displaystyle\int \frac{x}{(x+1)^2}\,dx =$

(a) $\ln|x+1| + \frac{1}{x+1} + C$

(b) $\ln|x+1| - \frac{1}{x+1} + C$

(c) $\ln|x+1| + \frac{1}{3(x+1)^2} + C$

(d) $\ln|x+1| - \frac{1}{3(x+1)^2} + C$

(e) $\ln|x+1| + \frac{3}{(x+1)^2} + C$

$u = x+1 \qquad x = u-1$

$du = dx$

$$\int \frac{u-1}{u^2}\,du = \int\left(\frac{1}{u} - \frac{1}{u^2}\right)du$$
$$= \ln|u| + \frac{1}{u} + C$$
$$= \ln|x+1| + \frac{1}{x+1} + C$$

4. A spring has a natural length of 2 m. It requires 27 J of work to stretch the spring from 2 m to 5 m. How much work is done in stretching the spring from 3 m to 4 m?

(a) $\frac{27}{2}$ J

(b) 21 J

(c) $\frac{63}{2}$ J

(d) 6 J

(e) 9 J

Given : $\int_0^3 kx\,dx = 27$

$k\frac{x^2}{2}\Big|_0^3 = 27$

$\frac{k}{2}(9) = 27$

$k = 6$

$f(x) = 6x$

$W = \int_1^2 6x\,dx$

$= 3x^2\Big|_1^2$

$= 9\text{ J}$

5. Consider the region bounded by $x = 1 - y^2$ and the y axis. Find the volume of the solid obtained by rotating the region about the y axis.

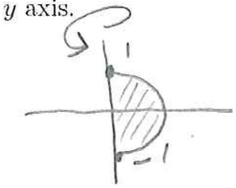
(a) $\frac{8\pi}{15}$

(b) $\frac{32\pi}{15}$

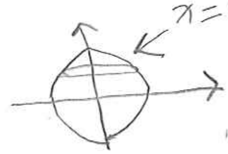
(c) $\frac{16\pi}{15}$

(d) $\frac{4\pi}{3}$

(e) $\frac{8\pi}{5}$



$x = 1 - y^2$



disk method

$V = \int_{-1}^{1} \pi(1-y^2)^2\,dx$

using symmetry $2\pi\int_0^1 (1 - 2y^2 + y^4)\,dy$

$2\pi\left(y - \frac{2y^3}{3} + \frac{y^5}{5}\right)\Big|_0^1$

$2\pi\left(1 - \frac{2}{3} + \frac{1}{5}\right)$

$2\pi\left(\frac{15 - 10 + 3}{15}\right) = 2\pi\left(\frac{8}{15}\right)$

$= \frac{16\pi}{15}$

6. $\int \sin^6 x \cos^3 x\,dx =$

(a) $-\frac{\sin^7 x}{7} + \frac{\sin^9 x}{9} + C$

(b) $\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C$

(c) $\frac{\sin^7 x}{7} + \frac{\sin^9 x}{9} + C$

(d) $\frac{\sin^7 x}{6} - \frac{\sin^9 x}{8} + C$

(e) $6\sin^5 x - 8\sin^7 x + C$

$\int \sin^6 x \cos^2 x \cos x\,dx$

$= \int \sin^6 x(1 - \sin^2 x)\cos x\,dx$

$u = \sin x$, so $du = \cos x\,dx$

$= \int u^6(1-u^2)\,du = \int (u^6 - u^8)\,du$

$= \frac{u^7}{7} - \frac{u^9}{9} + C$

$= \frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C$

7. A 10 foot long cable that weighs 60 pounds is hanging from a roof. At the end of this cable, there is a 20 pound box. How much work is done in lifting the entire cable and box to the top of the roof?

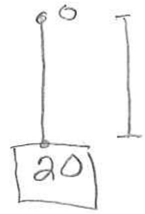
(a) 400 foot pounds

(b) 1100 foot pounds

(c) $\frac{2375}{3}$ foot pounds

(d) 200 foot pounds

(e) 500 foot pounds



$$W = \int_0^{10} \left((10-y)(6) + 20\right) dy$$

$$= \int_0^{10} (80-6y)\, dy$$

$$= (80y - 3y^2)\Big|_0^{10}$$

$$= 800 - 300$$

$$= 500 \text{ ft-lbs}$$

8. $\displaystyle\int_0^1 xe^{5x}\, dx =$

(a) $\dfrac{1+4e^5}{25}$

(b) $\dfrac{1+6e^5}{25}$

(c) $\dfrac{1+5e^5}{25}$

(d) $\dfrac{4}{25}e^5$

(e) $\dfrac{1+5e^5}{5}$

$u = x \qquad dv = e^{5x}\,dx$

$du = dx \qquad v = \frac{1}{5}e^{5x}$

$$\int_0^1 xe^{5x}\,dx = \frac{x}{5}e^{5x}\Big|_0^1 - \int_0^1 \frac{1}{5}e^{5x}\,dx$$

$$= \left(\frac{x}{5}e^{5x} - \frac{1}{25}e^{5x}\right)\Big|_0^1$$

$$= \frac{1}{5}e^5 - \frac{1}{25}e^5 + \frac{1}{25} = \frac{4e^5+1}{25}$$

9. For what value of b is the average value of $f(x) = 6x - 1$ on the interval $[0, b]$ equal to 7?

(a) $b = 2$

(b) $b = \frac{16}{3}$

(c) $b = 8$

(d) $b = \frac{8}{3}$

(e) $b = \frac{4}{3}$

$$\frac{1}{b}\int_0^b (6x-1)\,dx = 7$$

$$\frac{1}{b}(3x^2 - x)\Big|_0^b = 7$$

$$\frac{1}{b}(3b^2 - b) = 7$$

$3b - 1 = 7$, so $b = \frac{8}{3}$

10. $\displaystyle\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx =$

(a) $\frac{2}{21}$

(b) $\frac{2}{35}$

(c) $-\frac{2}{35}$

(d) $\frac{10}{21}$

(e) $\frac{12}{35}$

$$\int_0^{\frac{\pi}{4}} \tan^2 x \sec^2 x \sec^2 x \, dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^2 x (\tan^2 x + 1) \sec^2 x \, dx$$

$$u = \tan x \quad \begin{cases} x = \frac{\pi}{4}, & u = 1 \\ x = 0, & u = 0 \end{cases}$$

$$du = \sec^2 x \, dx$$

$$= \int_0^1 u^2 (u^2 + 1) \, du = \int_0^1 (u^4 + u^2) \, du$$

$$= \left(\frac{u^5}{5} + \frac{u^7}{7} \right) \Big|_0^1$$

$$= \frac{1}{5} + \frac{1}{7} = \frac{12}{35}$$

11. $\displaystyle\int_0^2 |x^2 - 1| \, dx =$

(a) 2

(b) $\frac{2}{3}$

(c) 4

(d) $\frac{4}{3}$

(e) 6

$$\int_0^1 (1 - x^2) \, dx + \int_1^2 (x^2 - 1) \, dx$$

$$\left(x - \frac{x^3}{3} \right) \Big|_0^1 + \left(\frac{x^3}{3} - x \right) \Big|_1^2$$

$$1 - \frac{1}{3} + \frac{8}{3} - 2 - \frac{1}{3} + 1 = 2$$

12. Find the area bounded by $y = \sqrt{x}$, $y = x$, $x = 0$, $x = 4$.

(a) $\frac{8}{3}$

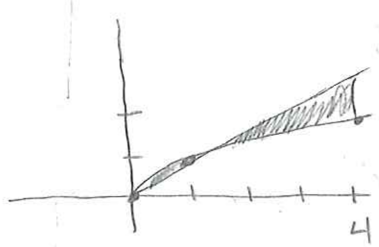
(b) 3

(c) $\frac{5}{3}$

(d) 4

(e) $\frac{7}{3}$

curves intersect at $(1,1)$ & $(0,0)$



$$A = \int_0^1 (\sqrt{x} - x) \, dx + \int_1^4 (x - \sqrt{x}) \, dx$$

$$A = \left(\frac{2}{3} x^{\frac{3}{2}} - \frac{x^2}{2} \right) \Big|_0^1 + \left(\frac{x^2}{2} - \frac{2}{3} x^{\frac{3}{2}} \right) \Big|_1^4$$

$$A = \frac{2}{3} - \frac{1}{2} + 8 - \frac{2}{3} \cdot 8 - \frac{1}{2} + \frac{2}{3}$$

$$A = 3$$

13. $\int_1^{e^3} \ln x \, dx =$

(a) $\frac{1}{e^3} - 1$

(b) $-\frac{2}{3}$

(c) $2e^3 - 1$

(d) $\frac{2}{3}$

(e) $8 + 16e^3$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$x \ln x \Big|_1^{e^3} - \int_1^{e^3} x \cdot \frac{1}{x} dx$

$(x \ln x - x)\Big|_1^{e^3}$

$e^3 \ln e^3 - e^3 - (-1)$

$3e^3 - e^3 + 1 = 2e^3 + 1$ *Question omitted*

14. $\int \frac{\sin(3x)}{\sqrt{1 - \cos(3x)}} dx =$

(a) $2\sqrt{1 - \cos(3x)} + C$

(b) $-2\sqrt{1 - \cos(3x)} + C$

(c) $-\frac{2}{3}\sqrt{1 - \cos(3x)} + C$

(d) $6\sqrt{1 - \cos(3x)} + C$

(e) $\frac{2}{3}\sqrt{1 - \cos(3x)} + C$

$u = 1 - \cos(3x)$

$du = 3\sin(3x)$

$\frac{1}{3}\int \frac{1}{\sqrt{u}} du = \frac{2}{3}\sqrt{u} + C$

$= \frac{2}{3}\sqrt{1 - \cos(3x)} + C$

15. Consider the region bounded by $y = \sqrt[3]{x}$ and $y = x^2$. Find the volume of the solid obtained by rotating the region about the x axis.

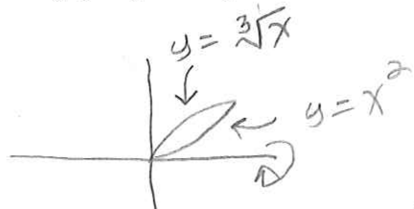
(a) $\frac{7\pi}{10}$

(b) $\frac{\pi}{5}$

(c) $\frac{8\pi}{5}$

(d) $\frac{2\pi}{5}$

(e) $\frac{9\pi}{10}$



washers: $V = \int_0^1 \pi(x^{\frac{2}{3}} - x^4) dx$

$= \pi\left(\frac{3}{5}x^{\frac{5}{3}} - \frac{x^5}{5}\right)\Big|_0^1$

$= \pi\left(\frac{3}{5} - \frac{1}{5}\right) = \frac{2\pi}{5}$

PART II: Work Out

16. (8 pts) Find $\int e^x \cos(2x)\,dx$.

Let $u = e^x$ and $dv = \cos(2x)\,dx$

$du = e^x dx \quad v = \frac{1}{2}\sin(2x)$

$$\int e^x \cos(2x)\,dx = \frac{1}{2} e^x \sin(2x) - \frac{1}{2}\underbrace{\int e^x \sin(2x)\,dx}$$

now, do parts again for $\int e^x \sin(2x)\,dx$

Let $u = e^x \quad dv = \sin(2x)\,dx$

$du = e^x dx \quad v = -\frac{1}{2}\cos(2x)$

so, $\int e^x \sin(2x)\,dx = \boxed{-\frac{1}{2} e^x \cos(2x) + \frac{1}{2}\int e^x \cos(2x)\,dx}$

$$\int e^x \cos(2x)\,dx = \frac{1}{2} e^x \sin(2x) - \frac{1}{2}\left(-\frac{1}{2} e^x \cos(2x) + \frac{1}{2}\int e^x \cos(2x)\,dx\right)$$

$$= \frac{1}{2} e^x \sin(2x) + \frac{1}{4} e^x \cos(2x) - \underbrace{\frac{1}{4}\int e^x \cos(2x)\,dx}_{\text{add to other side}}$$

$$\frac{5}{4}\int e^x \cos(2x)\,dx = \frac{1}{2} e^x \sin(2x) + \frac{1}{4} e^x \cos(2x)$$

$$\int e^x \cos(2x)\,dx = \frac{4}{5}\left(\frac{1}{2} e^x \sin(2x) + \frac{1}{4} e^x \cos(2x) + C\right)$$

17. (8 pts) Find $\int x^2 \tan^2(x^3)\,dx$.

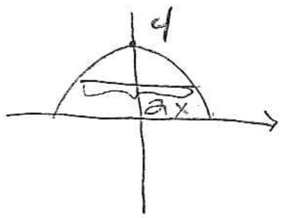
u- subs where $u = x^3$, so $du = 3x^2 dx$

$$\frac{1}{3}\int \tan^2 u\,du = \frac{1}{3}\int (\sec^2 u - 1)\,du$$

$$= \frac{1}{3}(\tan u - u) + C$$

$$= \frac{1}{3}(\tan(x^3) - x^3) + C$$

18. (9 pts) Find the volume of the solid S whose base is the region bounded by $y = 4 - x^2$ and the x axis and whose cross sections perpendicular to the **y axis** are squares.



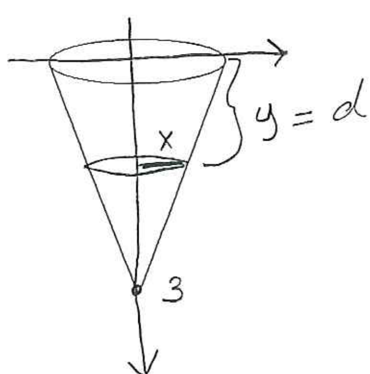
$$V = \int_0^4 (\text{side})^2\,dy = \int_0^4 4(4-y)\,dy$$

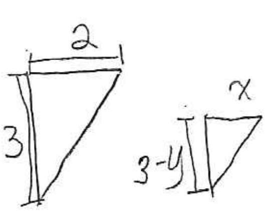
$$= (16y - 2y^2)\Big|_0^4$$

$$= \boxed{32}$$

side of square $= 2x = 2\sqrt{4-y}$

19. (9 pts) An inverted conical shaped tank is full of water as shown below. If the cone has height 3 m and radius 2 m, find the work done in pumping all of the water out of the cone. Note: the weight density of water is $\rho g = 9800$ Newtons per cubic meter.





$$\frac{2}{3} = \frac{x}{3-y}$$

$$x = \frac{2}{3}(3-y)$$

$V_{slice} = \pi r^2\,dy$, where $r = \frac{2}{3}(3-y)$

$$= \pi \cdot \frac{4}{9}(3-y)^2\,dy$$

$$F_{slice} = \frac{4}{9}\pi \rho g (3-y)^2\,dy$$

$$W_{slice} = \frac{4}{9}\pi \rho g (3-y)^2 (y)\,dy$$

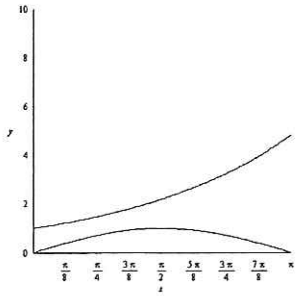
$$W = \int_0^3 \frac{4}{9}\pi\rho g\,(9 - 6y + y^2)\,y\,dy$$

$$= \frac{4\pi\rho g}{9}\int_0^3 (9y - 6y^2 + y^3)\,dy$$

$$= \frac{4\pi\rho g}{9}\left(\frac{9y^2}{2} - 2y^3 + \frac{y^4}{4}\right)\Big|_0^3$$

$$= \frac{4\pi\rho g}{9}\left(\frac{81}{2} - 54 + \frac{81}{4}\right) J = \boxed{3\rho g\,J}$$

20. Consider the region R bounded by $y = e^{x/2}$, $y = \sin x$, $x = 0$ and $x = \pi$.



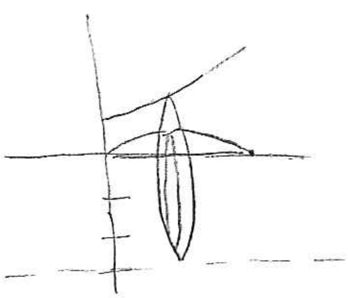
a.) (7 pts) Find the area of R.

$$A = \int_0^{\pi} \left(e^{\frac{x}{2}} - \sin x \right) dx$$

$$= \left(2e^{\frac{x}{2}} + \cos x \right) \Big|_0^{\pi}$$

$$= 2e^{\frac{\pi}{2}} - 1 - (2+1) \quad \text{or} \quad 2e^{\frac{\pi}{2}} - 4$$

b.) (7 pts) Set up the integral that gives the volume of the solid obtained by rotating R about the **horizontal** line $y = -3$. **Do not evaluate the integral.**

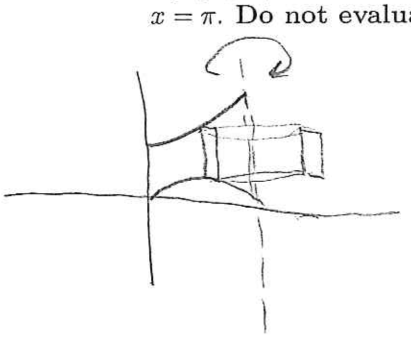


washers $R = e^{\frac{x}{2}} + 3$

$r = \sin x + 3$

$$V = \int_0^{\pi} \pi \left(\left(e^{\frac{x}{2}} + 3\right)^2 + (\sin x + 3)^2 \right) dx$$

c.) (7 pts) Set up the integral that gives the volume of the solid obtained by rotating R about the **vertical line** $x = \pi$. **Do not evaluate the integral.**



shells: $V = \int_0^{\pi} 2\pi(\pi - x)\left(e^{\frac{x}{2}} - \sin x \right) dx$

$r = \pi - x$

$h = e^{\frac{x}{2}} - \sin x$