MATH 152 Spring 2016
COMMON EXAM I - VERSION B

LAST NAME: Key FIRST NAME: ______________________

INSTRUCTOR: ______________________

SECTION NUMBER: ______________

UIN: ______________________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 4 points.
4. In Part 2 (Problems 16-20), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		8
17		8
18		9
19		9
20		21
Total		100

PART I: Multiple Choice. 3 points each.

1. $\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx =$

   (a) $\frac{12}{35}$

   (b) $\frac{10}{21}$

   (c) $-\frac{2}{35}$

   (d) $\frac{2}{21}$

   (e) $\frac{2}{35}$

$\int_0^{\frac{\pi}{4}} \tan^4 x \sec^2 x \sec^2 x \, dx$

$\int_0^{\frac{\pi}{4}} \tan^4 x (\tan^2 x + 1) \sec^2 x \, dx$

$u = \tan x$
$du = \sec^2 x \, dx$

$= \int_0^1 u^4 (u^2 + 1) \, du$

$= \int_0^1 (u^6 + u^4) \, du$

$= \left(\frac{u^7}{7} + \frac{u^5}{5}\right)\Big|_0^1$

$= \frac{12}{35}$

2. A 10 foot long cable that weighs 60 pounds is hanging from a roof. At the end of this cable, there is a 20 pound box. How much work is done in lifting the entire cable and box to the top of the roof?

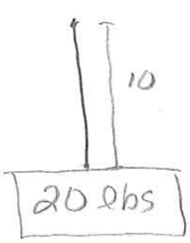
   (a) 1100 foot pounds

   (b) 200 foot pounds

   (c) 400 foot pounds

   (d) 500 foot pounds

   (e) $\frac{2375}{3}$ foot pounds



$W = \int_0^{10} (10 - y)(6) + 20 \, dy$

$= \int_0^{10} (80 - 6y) \, dy$

$= (80y - 3y^2)\Big|_0^{10}$

$= 800 - 300 = 500$ ft-lbs

3. Consider the region bounded by $y = \sqrt[3]{x}$ and $y = x^2$. Find the volume of the solid obtained by rotating the region about the x axis.

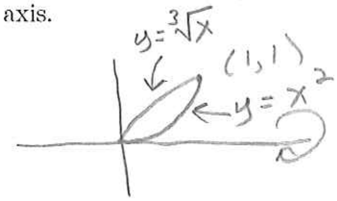
   (a) $\frac{2\pi}{5}$

   (b) $\frac{7\pi}{10}$

   (c) $\frac{9\pi}{10}$

   (d) $\frac{\pi}{5}$

   (e) $\frac{8\pi}{5}$



washers:

$V = \int_0^1 \pi (x^{\frac{2}{3}} - x^4) \, dx$

$= \pi \left(\frac{3}{5} y^{\frac{5}{3}} - \frac{y^5}{5}\right)\Big|_0^1$

$= \pi \left(\frac{2}{5}\right)$

4. $\int_0^1 xe^{5x}\,dx =$

(a) $\frac{4}{25}e^5$

(b) $\frac{1+5e^5}{5}$

(c) $\frac{1+6e^5}{25}$

(d) $\frac{1+4e^5}{25}$

(e) $\frac{1+5e^5}{25}$

$u = x \qquad dv = e^{5x}\,dx$

$du = dx \qquad v = \frac{1}{5}e^{5x}$

$\int_0^1 xe^{5x}\,dx = \frac{x}{5}e^{5x}\Big|_0^1 - \int_0^1 \frac{1}{5}e^{5x}\,dx$

$= \left(\frac{x}{5}e^{5x} - \frac{1}{25}e^{5x}\right)\Big|_0^1$

$= \frac{1}{5}e^5 - \frac{1}{25}e^5 + \frac{1}{25} = \frac{4e^5+1}{25}$

5. Find the area bounded by $y = e^x$, $y = e^{-x}$, $x = 0$ and $x = 1$.

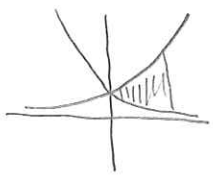
(a) $e + \frac{1}{e} + 2$

(b) $1 + \frac{1}{e}$

(c) $e - \frac{1}{e}$

(d) $1 + \frac{1}{e} - 2$

(e) $e + \frac{1}{e} - 2$



$A = \int_0^1 (e^x - e^{-x})\,dx$

$= (e^x + e^{-x})\Big|_0^1$

$= e + e^{-1} - 2$

$= e + \frac{1}{e} - 2$

6. $\int \sin^6 x \cos^3 x\,dx =$

(a) $6\sin^5 x - 8\sin^7 x + C$

(b) $\frac{\sin^7 x}{6} - \frac{\sin^9 x}{8} + C$

(c) $-\frac{\sin^7 x}{7} + \frac{\sin^9 x}{9} + C$

(d) $\frac{\sin^7 x}{7} + \frac{\sin^9 x}{9} + C$

(e) $\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C$

$\int \sin^6 x \cos^2 x \cos x\,dx$

$= \int \sin^6 x (1 - \sin^2 x) \cos x\,dx$

$u = \sin x \qquad du = \cos x\,dx$

$= \int u^6 (1 - u^2)\,du$

$= \int (u^6 - u^8)\,du = \frac{u^7}{7} - \frac{u^9}{9}$

$= \frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C$

7. $\int_1^{e^3} \ln x \, dx =$

(a) $-\frac{2}{3}$

(b) $2e^3 + 1$

(c) $\frac{1}{e^3} - 1$

(d) $8 + 16e^3$

(e) $\frac{2}{3}$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$x \ln x \Big|_1^{e^3} - \int_1^{e^3} x \cdot \frac{1}{x} dx$

$(x \ln x - x) \Big|_1^{e^3} = 3e^3 - e^3 + 1$

$= 2e^3 + 1$

8. Consider the region bounded by $x = 1 - y^2$ and the y axis. Find the volume of the solid obtained by rotating the region about the y axis.

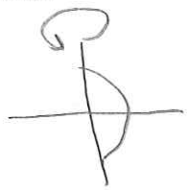
(a) $\frac{8\pi}{5}$

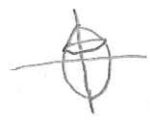
(b) $\frac{32\pi}{15}$

(c) $\frac{16\pi}{15}$

(d) $\frac{8\pi}{15}$

(e) $\frac{4\pi}{3}$





disk method

$V = \int_{-1}^{1} \pi r^2 dy$

use symmetry, where $r = 1 - y^2$

$V = 2\int_0^1 \pi (1-y^2)^2$

$= 2\pi \int_0^1 (1 - 2y^2 + y^4) dy$

$= 2\pi \left(y - \frac{2}{3}y^3 + \frac{1}{5}y^5 \right) \Big|_0^1 = \frac{16\pi}{15}$

9. For what value of b is the average value of $f(x) = 6x - 1$ on the interval $[0, b]$ equal to 7?

(a) $b = 8$

(b) $b = \frac{8}{3}$

(c) $b = 2$

(d) $b = \frac{4}{3}$

(e) $b = \frac{16}{3}$

$\frac{1}{b} \int_0^b (6x - 1) dx = 7$

$\frac{1}{b} (3x^2 - x) \Big|_0^b = 7$

$\frac{1}{b} (3b^2 - b) = 7$

$3b - 1 = 7$

$b = \frac{8}{3}$

10. Find the average value of $f(x) = \sin^2 x$ on the interval $\left[0, \frac{\pi}{8}\right]$.

(a) $\frac{4}{\pi}\left(\frac{\pi}{8} + \frac{\sqrt{2}}{4}\right)$

(b) $\frac{1}{2}\left(\frac{\pi}{8} + \frac{\sqrt{2}}{4}\right)$

(c) $\frac{4}{\pi}\left(\frac{\pi}{8} - \sqrt{2}\right)$

(d) $\frac{1}{2}\left(\frac{\pi}{8} - \frac{\sqrt{2}}{4}\right)$

(e) $\frac{4}{\pi}\left(\frac{\pi}{8} - \frac{\sqrt{2}}{4}\right)$

$$\frac{8}{\pi}\int_0^{\frac{\pi}{8}} \sin^2 x\,dx = \frac{8}{\pi}\cdot\frac{1}{2}\int_0^{\frac{\pi}{8}}(1-\cos(2x))\,dx$$

$$= \frac{4}{\pi}\left(x - \frac{1}{2}\sin(2x)\right)\Big|_0^{\frac{\pi}{8}}$$

$$= \frac{4}{\pi}\left(\frac{\pi}{8} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right)$$

$$= \frac{4}{\pi}\left(\frac{\pi}{8} - \frac{\sqrt{2}}{4}\right)$$

11. A spring has a natural length of 2 m. It requires 27 J of work to stretch the spring from 2 m to 5 m. How much work is done in stretching the spring from 3 m to 4 m?

(a) 21 J

(b) $\frac{63}{2}$ J

(c) $\frac{27}{2}$ J

(d) 9 J

(e) 6 J

Given: $\int_0^3 kx\,dx = 27$

$$k\frac{x^2}{2}\Big|_0^3 = 27$$

$\frac{9}{2}k = 27$ so $k = 6$

$$W = \int_1^2 6x\,dx = 3x^2\Big|_1^2 = 9\text{ J}$$

12. $\int \frac{x}{(x+1)^2}\,dx =$

(a) $\ln|x+1| + \frac{3}{(x+1)^2} + C$

(b) $\ln|x+1| + \frac{1}{3(x+1)^2} + C$

(c) $\ln|x+1| - \frac{1}{3(x+1)^2} + C$

(d) $\ln|x+1| - \frac{1}{x+1} + C$

(e) $\ln|x+1| + \frac{1}{x+1} + C$

$u = x+1$, so $x = u-1$

$du = dx$

$$\int \frac{x}{(x+1)^2} = \int \frac{u-1}{u^2}\,du$$

$$= \int\left(\frac{1}{u} - \frac{1}{u^2}\right)du$$

$$= \ln|u| + \frac{1}{u} + C$$

$$= \ln|x+1| + \frac{1}{x+1} + C$$

13. $\int \frac{\sin(3x)}{\sqrt{1-\cos(3x)}}\,dx =$

(a) $-\frac{2}{3}\sqrt{1-\cos(3x)}+C$

(b) $6\sqrt{1-\cos(3x)}+C$

(c) $2\sqrt{1-\cos(3x)}+C$

(d) $\frac{2}{3}\sqrt{1-\cos(3x)}+C$

(e) $-2\sqrt{1-\cos(3x)}+C$

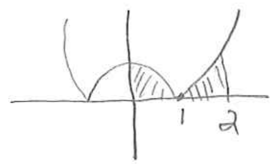
$u = 1-\cos(3x)$

$du = 3\sin(3x)\,dx$

$\frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{2}{3}\sqrt{u}+C$

$= \frac{2}{3}\sqrt{1-\cos(3x)}+C$

14. $\int_0^2 |x^2-1|\,dx =$



(a) 6

(b) $\frac{4}{3}$

(c) 2

(d) $\frac{2}{3}$

(e) 4

$\int_0^2 |x^2-1|\,dx = \int_0^1 (1-x^2)\,dx + \int_1^2 (x^2-1)\,dx$

$= \left(x - \frac{x^3}{3}\right)\Big|_0^1 + \left(\frac{x^3}{3} - x\right)\Big|_1^2$

$= 1 - \frac{1}{3} + \frac{8}{3} - 2 - \frac{1}{3} + 1$

$= 2$

15. Find the area bounded by $y=\sqrt{x}$, $y=x$, $x=0$, $x=4$.

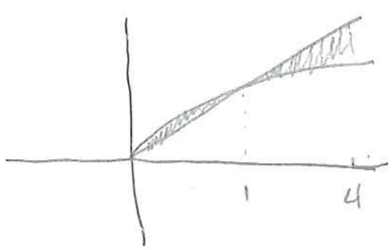
(a) $\frac{7}{3}$

(b) 3

(c) $\frac{8}{3}$

(d) 4

(e) $\frac{5}{3}$



$A = \int_0^1 (\sqrt{x} - x)\,dx + \int_1^4 (x - \sqrt{x})\,dx$

$= \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2}\right)\Big|_0^1 + \left(\frac{x^2}{2} - \frac{2}{3}x^{\frac{3}{2}}\right)\Big|_1^4$

$= \frac{2}{3} - \frac{1}{2} + 8 - \frac{2}{3}(8) - \frac{1}{2} + \frac{2}{3} = 3$

PART II: Work Out

16. (8 pts) Find $\int e^x \cos(4x)\, dx$.

Parts: $u = e^x \quad dv = \cos(4x)\,dx$

$du = e^x dx \quad v = \frac{1}{4}\sin(4x)$

$$\int e^x \cos(4x)\,dx = \frac{1}{4}e^x \sin(4x) - \frac{1}{4}\underbrace{\int e^x \sin(4x)\,dx}$$

now do parts again to find $\int e^x \sin(4x)\,dx$

$u = e^x; \quad dv = \sin(4x)\,dx$

$du = e^x\,dx, \quad v = -\frac{1}{4}\cos(4x)$

so, $\int e^x \sin(4x)\,dx = \boxed{-\frac{1}{4}e^x\cos(4x) + \frac{1}{4}\int e^x \cos(4x)\,dx}$

$$\int e^x \cos(4x)\,dx = \frac{1}{4}e^x \sin(4x) - \frac{1}{4}\left(-\frac{1}{4}e^x\cos(4x) + \frac{1}{4}\int e^x\cos(4x)\,dx\right)$$

$$= \frac{1}{4}e^x\sin(4x) + \frac{1}{16}e^x\cos(4x) + \underbrace{\frac{1}{16}\int e^x\cos(4x)\,dx}_{\text{add to other side}}$$

$$\frac{17}{16}\int e^x\cos(4x)\,dx = \frac{1}{4}e^x\sin(4x) + \frac{1}{16}e^x\cos(4x) + C$$

$$\int e^x\cos(4x)\,dx = \frac{16}{17}\left(\frac{1}{4}e^x\sin(4x) + \frac{1}{16}e^x\cos(4x) + C\right)$$

17. (8 pts) Find $\int x^2 \tan^2(x^3)\, dx$.

u-sub: $u = x^3$

$du = 3x^2 dx$

$$\int x^2 \tan^2(x^3)\,dx = \frac{1}{3}\int \tan^2 u\,du$$

$$= \frac{1}{3}\int(\sec^2 u - 1)\,du$$

$$= \frac{1}{3}(\tan u - u) + C$$

$$= \frac{1}{3}(\tan x^3 - x^3) + C$$

18. (9 pts) Find the volume of the solid S whose base is the region bounded by $y = 9 - x^2$ and the x axis and whose cross sections perpendicular to the **y axis** are squares.

$$V = \int_0^9 (\text{side})^2\, dy$$

$$= \int_0^9 4(9-y)\, dy$$

side $= 2x$

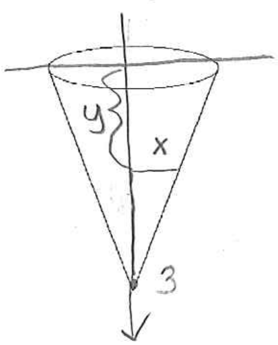
$= 2\sqrt{9-y}$

$$= \left(36y - 2y^2\right)\Big|_0^9$$

$$= 36(9) - 2(81)$$

$$= \boxed{162}$$

19. (9 pts) An inverted conical shaped tank is full of water as shown below. If the cone has height 3 m and radius 2 m, find the work done in pumping all of the water out of the cone. Note: the weight density of water is $\rho g = 9800$ Newtons per cubic meter.

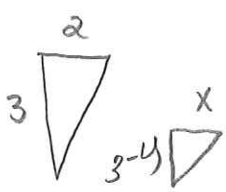


$V_{slice} = \pi r^2\, dy$, where $r = \frac{2}{3}(3-y)$

$V_{slice} = \pi \cdot \frac{4}{9}(3-y)^2\, dy$

$F_{slice} = \frac{4}{9}\pi \rho g (3-y)^2\, dy$

$W_{slice} = \frac{4}{9}\pi \rho g\, y(3-y)^2\, dy$



$$W = \int_0^3 \frac{4\pi}{9}\rho g\, y(9 - 6y + y^2)\, dy$$

$$= \frac{4\pi}{9}\rho g \int_0^3 (9y - 6y^2 + y^3)\, dy.$$

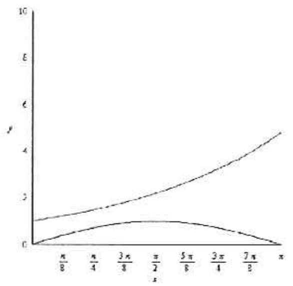
$\frac{x}{3-y} = \frac{2}{3}$

$x = \frac{2}{3}(3-y)$

$$= \frac{4\pi}{9}\rho g\left(\frac{9y^2}{2} - 2y^3 + \frac{y^4}{4}\right)\Big|_0^3$$

$$= \frac{4\pi}{9}\rho g\left(\frac{81}{2} - 54 + \frac{81}{4}\right) J \quad \text{or } 3\rho g J$$

20. Consider the region R bounded by $y = e^{x/2}$, $y = \sin x$, $x = 0$ and $x = \pi$.



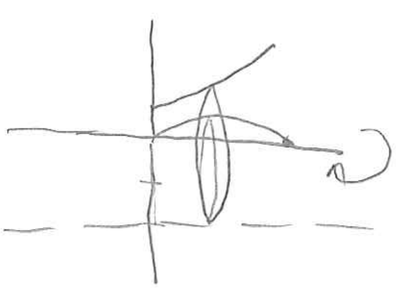
a.) (7 pts) Find the area of R.

$$A = \int_0^{\pi} \left(e^{\frac{x}{2}} - \sin x\right) dx$$

$$= \left(2e^{\frac{x}{2}} + \cos x\right) \Big|_0^{\pi}$$

$$= 2e^{\frac{\pi}{2}} - 1 - (2+1) = 2e^{\frac{\pi}{2}} - 4$$

b.) (7 pts) Set up the integral that gives the volume of the solid obtained by rotating R about the **horizontal** line $y = -2$. **Do not evaluate the integral.**



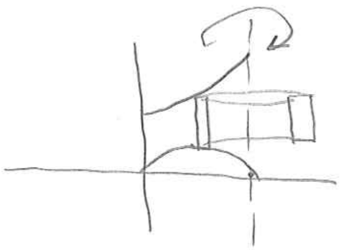
$y = -2$

washers: $R = e^{\frac{x}{2}} + 2$

$r = \sin x + 2$

$$V = \int_0^{\pi} \pi \left(\left(e^{\frac{x}{2}} + 2\right)^2 - (\sin x + 2)^2 \right) dx$$

c.) (7 pts) Set up the integral that gives the volume of the solid obtained by rotating R about the **vertical line** $x = \pi$. **Do not evaluate the integral.**



$x = \pi$

shells: $V = \int_0^{\pi} 2\pi r h \, dx$

$r = \pi - x$

$h = e^{\frac{x}{2}} - \sin x$

$$V = \int_0^{\pi} 2\pi(\pi - x)\left(e^{\frac{x}{2}} - \sin x\right) dx$$