MATH 152 Spring 2016
COMMON EXAM II - VERSION A

LAST NAME: ____________________ FIRST NAME: Key

INSTRUCTOR: ____________________

SECTION NUMBER: __________

UIN: ____________________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 4 points.
4. In Part 2 (Problems 16-20), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		5
17		8
18		12
19		10
20		9
21		11
Total		100

PART I: Multiple Choice. 3 points each.

1. Which of the following is the correct partial fraction decomposition for $f(x) = \dfrac{4x+3}{x^2(x^2-9)(x^2+4)}$?

(a) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3} + \dfrac{D}{x-3} + \dfrac{Ex+F}{x^2+4}$

(b) $\dfrac{A}{x^2} + \dfrac{B}{x+3} + \dfrac{C}{x-3} + \dfrac{Dx+E}{x^2+4}$

(c) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx}{x^2-9} + \dfrac{Ex}{x^2+4}$

(d) $\dfrac{A}{x^2} + \dfrac{Bx+C}{x^2-9} + \dfrac{Dx+E}{x^2+4}$

(e) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3} + \dfrac{D}{x-3} + \dfrac{E}{x+2} + \dfrac{F}{(x+2)^2}$

$x^2(x+3)(x-3)(x^2+4)$

x^2 repeating linear factor

$(x+3)$ & $(x-3)$ non repeating linear

x^2+4 irreducible quadratic

2. Find the length of the curve $y = \sqrt[3]{x^2}$, $0 \le y \le 1$.

(a) $\dfrac{2}{3}\left(\left(\dfrac{5}{3}\right)^{3/2} - 1\right)$

(b) $\dfrac{8}{27}\left(\left(\dfrac{13}{4}\right)^{3/2} - 1\right)$

(c) $\dfrac{2}{3}\left(\left(\dfrac{13}{4}\right)^{3/2} - 1\right)$

(d) $\dfrac{4}{9}\left(\left(\dfrac{5}{3}\right)^{3/2} - 1\right)$

(e) $\dfrac{3}{2}\left(\left(\dfrac{13}{4}\right)^{3/2} - 1\right)$

$y = x^{2/3}$, so $x = y^{\frac{3}{2}}$

$\dfrac{dx}{dy} = \dfrac{3}{2} y^{\frac{1}{2}} = \dfrac{3}{2}\sqrt{y}$

$L = \int_0^1 \sqrt{1 + \frac{9}{4}y}$ u-sub $u = 1 + \frac{9}{4}y$

$L = \frac{4}{9}\cdot\frac{2}{3}\left(1 + \frac{9}{4}y\right)^{3/2}\Big|_0^1$

$= \frac{8}{27}\left(\left(\frac{13}{4}\right)^{3/2} - 1\right)$

3. The integral $\displaystyle\int_{-1}^{2} \frac{dx}{x^3}$

(a) converges to $\dfrac{3}{8}$

(b) converges to $\dfrac{7}{32}$

(c) converges to $\dfrac{3}{4}$

(d) diverges

(e) converges to $\dfrac{3}{2}$

$\int_{-1}^{2} \frac{dx}{x^3}$ diverges since both $\int_{-1}^{0} \frac{dx}{x^3}$ and $\int_0^2 \frac{dx}{x^3}$ diverge

$\int_0^2 \frac{dx}{x^3} = \lim_{t\to 0^+} \int_t^2 \frac{dx}{x^3}$

$= \lim_{t\to 0^+} \frac{-1}{2x^2}\Big|_t^2$

$= \lim_{t\to 0^+}\left(-\frac{1}{8} + \frac{1}{2t^2}\right) = \infty$

4. Which of the following integrals results after performing an appropriate trigonometric substitution for $\int_0^{1/2} x^2\sqrt{1+4x^2}\,dx$?

(a) $8\int_0^{\pi/4} \tan^2\theta\sec^3\theta\,d\theta$

(b) $\frac{1}{4}\int_0^{\pi/2} \sin^2\theta\cos^2\theta\,d\theta$

(c) $\frac{1}{8}\int_0^{\pi/4} \tan^2\theta\sec^3\theta\,d\theta$

(d) $\frac{1}{8}\int_0^{\pi/4} \tan^2\theta\sec\theta\,d\theta$

(e) $4\int_0^{\pi/2} \sin^2\theta\cos^2\theta\,d\theta$

$2x = \tan\theta$ $\left\langle \begin{array}{l} x=\frac{1}{2},\ \theta=\frac{\pi}{4} \\ x=0,\ \theta=0 \end{array}\right.$

$x = \frac{1}{2}\sec^2\theta\,d\theta$

$\int_0^{\frac{\pi}{4}} \frac{1}{4}\tan^2\theta\sqrt{1+\tan^2\theta}\,\frac{1}{2}\sec^2\theta\,d\theta$

$\frac{1}{8}\int_0^{\frac{\pi}{4}} \tan^2\theta\sqrt{\sec^2\theta}\,\sec^2\theta\,d\theta$

$\frac{1}{8}\int_0^{\frac{\pi}{4}} \tan^2\theta\sec^3\theta\,d\theta$

5. Compute $\int_2^3 \frac{x^3}{x-1}\,dx$.

(a) $\frac{5}{2}+\ln 2$

(b) $\frac{59}{6}-\ln 2$

(c) $\frac{29}{6}+\ln 2$

(d) $\frac{29}{6}-\ln 2$

(e) $\frac{59}{6}+\ln 2$

Long division: $x-1 \overline{)\,x^3}$ quotient x^2+x+1

$x^3 - x^2$

x^2

$x^2 - x$

x

$x-1$

1

$\int_2^3 \left(x^2+x+1+\frac{1}{x-1}\right)dx$

$\left(\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x-1|\right)\Big|_2^3$

$=\frac{59}{6}+\ln 2$

6. The sequence $a_n = 2\ln(7n+3)-\ln(5n^2+1)$

(a) converges to $\ln\left(\frac{49}{5}\right)$

(b) converges to $\ln\left(\frac{7}{25}\right)$

(c) converges to $\ln\left(\frac{7}{5}\right)$

(d) converges to $\ln\left(\frac{5}{49}\right)$

(e) diverges

$\lim_{n\to\infty}\left(\ln(7n+3)^2-\ln(5n^2+1)\right)$

$\lim_{n\to\infty}\ln\left(\frac{(7n+3)^2}{5n^2+1}\right)$

$=\ln\left(\frac{49}{5}\right)$

7. $\sum_{n=0}^{\infty} \frac{2^{2n}}{5^{n+1}} =$ $\sum_{n=0}^{\infty} \frac{4^n}{5^{n+1}} = \frac{\frac{1}{5}}{1-\frac{4}{5}}$

$= 1$

(a) $\frac{1}{3}$

(b) 4

(c) $\frac{4}{3}$

(d) 1

(e) $\frac{1}{9}$

8. The integral $\int_1^{\infty} \frac{dx}{\sqrt{x}+e^{9x}}$ use comparison test

(a) converges by comparison with $\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$

(b) diverges by comparison with $\int_1^{\infty} \frac{1}{e^{9x}}\,dx$

(c) diverges by comparison with $\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$

(d) converges by comparison with $\int_1^{\infty} \frac{1}{e^{9x}}\,dx$

(e) converges to 0

$0 < \int_1^{\infty} \frac{dx}{\sqrt{x}+e^{9x}} < \int_1^{\infty} \frac{dx}{e^{9x}} = \int_1^{\infty} e^{-9x}\,dx$

$\lim_{t\to\infty} \left(\frac{-1}{9} \cdot \frac{1}{e^{9x}}\right)\Big|_1^t$

$= \lim_{t\to\infty} \frac{-1}{9}\left(\frac{1}{e^{9t}} - \frac{1}{e^9}\right)$

$= \frac{1}{9e^9} < \infty$

larger integral converges, so does smaller

9. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{2n+3}{n+5}$, find a_3 as well as the sum, S, of the series $\sum_{n=1}^{\infty} a_n$.

(a) $a_3 = \frac{1}{8}$ and the series diverges.

(b) $a_3 = \frac{1}{8}$ and $S = 2$.

(c) $a_3 = \frac{9}{8}$ and $S = 1$.

(d) $a_3 = \frac{9}{8}$ and the series diverges.

(e) $a_3 = \frac{71}{24}$ and $S = 2$.

$a_3 = S_3 - S_2$

$= \frac{9}{8} - \frac{7}{7}$

$= \frac{1}{8}$

$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{2n+3}{n+5}$

$= 2$

10. Which of the following integrals gives the surface area obtained by rotating the curve $y = \sin\left(\frac{x}{2}\right)$, $0 \le x \le \frac{\pi}{2}$ about the y axis?

(a) $\int_0^{\sqrt{2}/2} 2\pi y\sqrt{1+\frac{4}{1-y^2}}\,dy$

(b) $\int_0^{\sqrt{2}/2} 4\pi \arcsin y\sqrt{1+\frac{4}{1-y^2}}\,dy$

(c) $\int_0^{1} 4\pi \arcsin y\sqrt{1+\frac{4}{1-y^2}}\,dy$

(d) $\int_0^{1} 2\pi y\sqrt{1+\frac{4}{1-y^2}}\,dy$

(e) $\int_0^{\sqrt{2}/2} 2\pi \arcsin y\sqrt{1+\frac{4}{1-y^2}}\,dy$

$y = \sin\left(\frac{x}{2}\right)$

$\arcsin y = \frac{x}{2}$

$x = 2\arcsin y$

$\frac{dx}{dy} = \frac{2}{\sqrt{1-y^2}}$

@ y-axis

so $r = x$
$= 2\arcsin y$

$0 \le x \le \frac{\pi}{2}$

$0 \le y \le \frac{\sqrt{2}}{2}$

$SA = \int_0^{\frac{\sqrt{2}}{2}} 2\pi \cdot 2\arcsin y\sqrt{1+\frac{4}{1-y^2}}\,dy$

11. Given the sequence $a_1 = 1$ and $a_{n+1} = \sqrt{12+a_n}$ is increasing and bounded, what statement is true about a_n?

(a) converges to 4

(b) converges to 3

(c) converges to 2

(d) converges to 6

(e) diverges

Since $\{a_n\}$ is increasing & bounded,

$\lim_{n\to\infty} a_n = L$ exists.

$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\sqrt{12+a_n}$

$L = \sqrt{12+L}$

$L^2 = 12+L$

$L^2 - L - 12 = 0$

$(L+3)(L-4) = 0$

$L = 4$ since $a_1 = 1$ and a_n is increasing

12. Find s_4, the fourth partial sum, of the series $\sum_{n=1}^{\infty} \cos\left(\frac{n\pi}{3}\right)$.

(a) $s_4 = -\frac{3}{2}$

(b) $s_4 = -\frac{1}{2}$

(c) $s_4 = -1-\frac{\sqrt{3}}{2}$

(d) $s_4 = \frac{1}{2}$

(e) $s_4 = -\frac{\sqrt{3}}{2}$

$S_4 = \cos\frac{\pi}{3} + \cos\frac{2\pi}{3} + \cos\pi + \cos\frac{4\pi}{3}$

$= \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} - 1 - \frac{1}{2}$

$= -\frac{3}{2}$

13. After making an appropriate trigonometric substitution, which of the following integrals is equivalent to $\int \sqrt{-x^2+2x+3}\,dx$?

$$-x^2+2x+3 = -(x^2-2x)+3$$
$$= -(x^2-2x+1)+4$$
$$= -(x-1)^2+4$$

(a) $4\int \sec\theta \tan^2\theta \, d\theta$

(b) $2\int \cos\theta \, d\theta$

(c) $4\int \sec^3\theta \, d\theta$

(d) $4\int \cos^2\theta \, d\theta$

(e) $2\int \sec\theta \, d\theta$

let $x-1 = 2\sin\theta$
$dx = 2\cos\theta \, d\theta$

$$\int \sqrt{4-(x-1)^2}\,dx = \int \sqrt{4-4\sin^2\theta}\cdot 2\cos\theta\,d\theta$$
$$= 4\int \cos^2\theta\,d\theta$$

14. Find the length of the curve $x = \frac{t^2}{2}$, $y = \frac{t^3}{3}$, $0 \le t \le 1$.

(a) $\frac{4}{3}\left(2^{3/2}-1\right)$

(b) $\frac{1}{54}\left(10^{3/2}-1\right)$

(c) $3(2^{3/2}-1)$

(d) $\frac{1}{24}\left(10^{3/2}-1\right)$

(e) $\frac{1}{3}\left(2^{3/2}-1\right)$

$\frac{dx}{dt} = t$, $\frac{dy}{dt} = t^2$

$$L = \int_0^1 \sqrt{t^2+t^4}\,dt$$
$$= \int_0^1 \sqrt{t^2(1+t^2)}\,dt$$
$$= \int_0^1 t\sqrt{1+t^2}\,dt$$

u-sub
$u = 1+t^2$

$$= \frac{1}{2}\cdot\frac{2}{3}(1+t^2)^{\frac{3}{2}}\Big|_0^1$$
$$= \frac{1}{3}\left(2^{3/2}-1\right)$$

15. Given the sequence $a_n = \frac{\ln n}{n}$, $n \ge 5$, which of the following statements are true?

I. a_n is decreasing II. $(-1)^n a_n$ converges to 0 III. a_n is bounded

(a) only I. and II.

(b) only II. and II.

(c) only I. and III.

(d) only II.

(e) All of the above statements are true.

since $\frac{d}{dx}\frac{\ln x}{x} = \frac{\frac{1}{x}\cdot x - \ln x}{x^2}$

$$= \frac{1-\ln x}{x^2} < 0 \quad \text{for } x > e$$

$\left\{\frac{\ln n}{n}\right\}$ decreases.

$\lim_{n\to\infty} \frac{\ln n}{n} \overset{L}{=} \lim_{n\to\infty} \frac{1}{n} = 0$, thus $\left\{\frac{(-1)^n \ln n}{n}\right\}$ converges to 0.

$\left\{\frac{\ln n}{n}\right\}$ is bounded between 0 & $\frac{\ln 5}{5}$

PART II: Work Out

16. Consider the sequence $a_n = \dfrac{3n}{5n+4}$.

a.) (2 pts) Find the limit of a_n.

$$\lim_{n\to\infty} \frac{3n}{5n+4} \overset{L}{=} \lim_{n\to\infty} \frac{3}{5} \quad \frac{\infty}{\infty}$$

$$= \frac{3}{5}$$

b.) (3 pts) Find the sum of the series $\sum_{n=1}^{\infty} a_n$ or explain why it diverges.

$\sum_{n=1}^{\infty} \frac{3n}{5n+4}$ diverges since $\lim_{n\to\infty} a_n = \frac{3}{5} \neq 0$

Test for divergence

17. (8 pts) Compute $\int_e^{\infty} \frac{\ln x}{x^2}\,dx$ or show that it diverges.

$u = \ln x \qquad dv = \frac{dx}{x^2}$

$du = \frac{dx}{x} \qquad v = -\frac{1}{x}$

$$\int \frac{\ln x}{x^2} = uv - \int v\,du$$

$$= -\frac{\ln x}{x} + \int \frac{dx}{x^2}$$

$$= -\frac{\ln x}{x} - \frac{1}{x}$$

$$\int_e^{\infty} \frac{\ln x}{x^2} = \lim_{t\to\infty} \int_e^t \frac{\ln x}{x^2} = \lim_{t\to\infty} \left(-\frac{\ln x}{x} - \frac{1}{x}\right)\Big|_e^t$$

$$= \lim_{t\to\infty} -\frac{\ln t}{t} - \frac{1}{t} - \left(-\frac{\ln e}{e} - \frac{1}{e}\right)$$

$$\lim_{t\to\infty} \frac{\ln t}{t} \overset{L}{=} \lim_{t\to\infty} \frac{\frac{1}{t}}{t}$$

$$= 0$$

$$= 0 - 0 + \frac{1}{e} + \frac{1}{e}$$

$$= \boxed{\frac{2}{e}}$$

18. a.) (4 pts) Find the partial fraction decomposition for $\frac{-2}{(2n+1)(2n-1)}$.

$$\frac{-2}{(2n+1)(2n-1)} = \frac{A}{2n+1} + \frac{B}{2n-1}$$

$$-2 = A(2n-1) + B(2n+1)$$

$n = \frac{1}{2}$: $-2 = B(2)$ $B = -1$

$n = -\frac{1}{2}$: $-2 = A(-2)$ $A = 1$

$$\frac{-2}{(2n+1)(2n-1)} = \frac{1}{2n+1} - \frac{1}{2n-1}$$

b.) (4 pts) Find a formula for s_n, the nth partial sum of the series $\sum_{n=1}^{\infty} \frac{-2}{(2n+1)(2n-1)}$. $= \sum_{n=1}^{\infty}\left(\frac{1}{2n+1} - \frac{1}{2n-1}\right)$

$$S_n = a_1 + a_2 + \dots + a_n$$

$$S_n = \underbrace{\frac{1}{3} - 1}_{a_1} + \underbrace{\frac{1}{5} - \frac{1}{3}}_{a_2} + \underbrace{\frac{1}{7} - \frac{1}{5}}_{a_3} + \dots + \underbrace{\frac{1}{2n-1} - \frac{1}{2n-3}}_{a_{n-1}} + \underbrace{\frac{1}{2n+1} - \frac{1}{2n-1}}_{a_n}$$

$$\boxed{S_n = -1 + \frac{1}{2n+1}}$$

c.) (4 pts) Find $\sum_{n=1}^{\infty} \frac{-2}{(2n+1)(2n-1)}$.

$$\sum_{n=1}^{\infty} \frac{-2}{(2n+1)(2n-1)} = \lim_{n\to\infty} S_n$$

$$= \lim_{n\to\infty}\left(-1 + \frac{1}{2n+1}\right)$$

$$= -1$$

19. Consider the surface obtained by rotating the curve $y = \ln(2x + 5)$, $1 \le x \le 2$, about the x-axis.

a.) (5 pts) Set up but do not evaluate an integral in terms of x that gives the area of the surface.

$$SA = \int_1^2 2\pi \ln(2x+5)\sqrt{1 + \frac{4}{(2x+5)^2}}\, dx$$

b.) (5 pts) Set up but do not evaluate an integral in terms of y that gives the area of the surface.

$1 \le x \le 2$

$\ln 7 \le y \le \ln 9$

$y = \ln(2x+5)$

$e^y = 2x+5$

$2x = e^y - 5$

$x = \frac{1}{2}(e^y - 5)$

$$SA = \int_{\ln 7}^{\ln 9} 2\pi y \sqrt{1 + \frac{1}{4}e^{2y}}\, dy$$

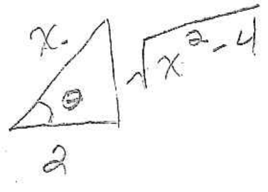
20. (9 pts) Find $\displaystyle\int \frac{1}{x^4\sqrt{x^2-4}}\, dx$. Express your answer without the use of trig or inverse trig functions.

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\, d\theta$

$$\int \frac{2\sec\theta\tan\theta}{16\sec^4\theta\sqrt{4\sec^2\theta - 4}}\, d\theta$$

$$\frac{1}{8}\int \frac{\tan\theta}{\sec^3\theta \cdot 2\tan\theta}\, d\theta$$



$$\frac{1}{16}\int \frac{d\theta}{\sec^3\theta}\, d\theta$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$\frac{1}{16}\int \cos^3\theta\, d\theta$$

$$\frac{1}{16}\int (1 - \sin^2\theta)\cos\theta\, d\theta$$

$$\frac{1}{16}\int (1-u^2)\, du = \frac{1}{16}\left(u - \frac{u^3}{3}\right) + C$$

$$\boxed{\frac{1}{16}\left(\frac{\sqrt{x^2-4}}{x} - \frac{1}{3}\left(\frac{\sqrt{x^2-4}}{x}\right)^3\right) + C} = \frac{1}{16}\left(\sin\theta - \frac{1}{3}\sin^3\theta\right) + C$$

21. (11 pts) Find $\int \frac{5x^2+7x+8}{(x-1)(x^2+9)}\,dx$

$$\frac{5x^2+7x+8}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$$5x^2+7x+8 = A(x^2+9)+(Bx+C)(x-1)$$

$$x=1 : 20 = A(10) \quad A=2$$

$$5x^2+7x+8 = 2x^2+18+Bx^2-Bx+Cx-C$$

$$= (2+B)x^2+(C-B)x-C+18$$

$$5 = 2+B$$

$$B=3$$

$$8 = -C+18$$

$$C=10$$

$$\int\left(\frac{2}{x-1}+\frac{3x-10}{x^2+9}\right)dx = \int\left(\frac{2}{x-1}+\frac{3x+10}{x^2+9}\right)dx$$

$$= \int\left(\frac{2}{x-1}+\frac{3x}{x^2+9}+\frac{10}{x^2+9}\right)dx$$

$$= 2\ln|x-1| + \frac{3}{2}\ln(x^2+9) + \frac{10}{3}\arctan\frac{x}{3}$$

$$+C$$