MATH 152 Spring 2016
COMMON EXAM III - VERSION A

LAST NAME: Answers FIRST NAME: ____________

INSTRUCTOR: ____________

SECTION NUMBER: ____________

UIN: ____________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-20), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		12
17		12
18		8
19		9
20		14
Total		100

PART I: Multiple Choice. 3 points each.

1. The region in $\mathbb{R}^3$ described by the equation $y = 8$ is:

   (a) A line parallel to the x-axis.
   (b) A plane parallel to the yz plane passing through the point $(0, 8, 0)$.
   (c) The point $(0, 8, 0)$.
   (d) A plane parallel to the xz plane passing through the point $(0, 8, 0)$. ANSWER
   (e) A line parallel to the z-axis.

2. Find the radius of convergence, R, and the interval of convergence, I, for $\sum_{n=1}^{\infty} \frac{(x+5)^n n!}{3^n}$.

   (a) $R = 3$, $I = (-8, -2)$
   (b) $R = 0$, $I = \{-5\}$ ANSWER
   (c) $R = \infty$, $I = \{-5\}$
   (d) $R = 3$, $I = (2, 8)$
   (e) $R = \infty$, $I = (-\infty, \infty)$

3. Find the third degree Taylor Polynomial, $T_3(x)$, for $f(x) = \cos(2x)$ at $a = \frac{\pi}{6}$.

   (a) $T_3(x) = \frac{1}{2} - \sqrt{3}\left(x - \frac{\pi}{6}\right) - \left(x - \frac{\pi}{6}\right)^2 + \frac{2\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right)^3$ ANSWER
   (b) $T_3(x) = \frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3$
   (c) $T_3(x) = \frac{1}{2} - \sqrt{3}\left(x - \frac{\pi}{6}\right) - 2\left(x - \frac{\pi}{6}\right)^2 + 4\sqrt{3}\left(x - \frac{\pi}{6}\right)^3$
   (d) $T_3(x) = \frac{\sqrt{3}}{2} - \left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right)^3$
   (e) $T_3(x) = \frac{1}{2} - \sqrt{3}\left(x - \frac{\pi}{6}\right) + \left(x - \frac{\pi}{6}\right)^2 + \frac{2\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right)^3$

4. Which of the following is a Maclaurin series for $f(x) = x^3 e^{-x^2}$?

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n)!}$

(b) $\sum_{n=0}^{\infty} \frac{x^{2n+3}}{n!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{n!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{5n}}{n!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{n!}$ ANSWER

5. Find the equation of the sphere whose diameter has endpoints $(2, 1, 6)$ and $(4, -3, 8)$.

(a) $(x+3)^2 + (y-1)^2 + (z+7)^2 = 6$
(b) $(x-3)^2 + (y+1)^2 + (z-7)^2 = 24$
(c) $(x-3)^2 + (y+1)^2 + (z-7)^2 = 6$ ANSWER
(d) $(x-3)^2 + (y-1)^2 + (z-7)^2 = 24$
(e) $(x-6)^2 + (y+2)^2 + (z-14)^2 = 24$

6. Write $f(x) = \frac{x^3}{8+x}$ as a power series centered at 0.

(a) $\sum_{n=0}^{\infty} \frac{x^{n+3}}{8^{n+1}}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3}}{64^n}$

(c) $\sum_{n=0}^{\infty} \left(-\frac{x}{8}\right)^{n+3}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3}}{8^{n+1}}$ ANSWER

(e) $\sum_{n=0}^{\infty} \left(-\frac{x}{8}\right)^{3n}$

7. Using the Alternating Series Estimation Theorem, what is the smallest number of terms we must use to approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ with error less than $\frac{1}{120}$?

(a) $n = 11$

(b) $n = 9$

(c) $n = 10$ ANSWER

(d) $n = 12$

(e) $n = 13$

8. Which of the following statements is true of the series $\sum_{n=1}^{\infty} \frac{n^3}{n^4 + n^2 + 1}$?

I. Diverges by comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

II. Diverges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

III. Diverges by Test for Divergence

(a) Only I and II are true

(b) Only I is true

(c) Only III is true

(d) Only II is true ANSWER

(e) None of I, II and III is true.

9. Find $f^{(23)}(3)$, that is, the 23^{rd} derivative of $f(x)$ at $x = 3$, if $f(x) = \sum_{n=0}^{\infty} \frac{(-3)^{n+2}}{(2n)!}(x-3)^n$.

(a) $\frac{(-3)^{24}(23)!}{(46)!}$

(b) 0

(c) $\frac{(-3)^{25}}{(46)!}$

(d) $\frac{(-3)^{25}(24)!}{(46)!}$

(e) $\frac{(-3)^{25}(23)!}{(46)!}$ ANSWER

10. $\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{2\pi}{3}\right)^{2n+1}}{(2n+1)!} =$

(a) $\frac{\sqrt{3}}{2}$ ANSWER

(b) $-\frac{1}{2}$

(c) $-\frac{\sqrt{3}}{2}$

(d) $\frac{1}{2}$

(e) $e^{2\pi/3}$

11. For which of the following series is the Ratio Test inconclusive?

I) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+3}$

II) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

III) $\sum_{n=1}^{\infty} n4^n$

(a) I only

(b) II only

(c) I and II only CORRECT

(d) II and III only

(e) I, II, and III

12. Suppose it is known that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -3$ and diverges when $x = 5$. Which of the following is certain to be true?

(a) $\sum_{n=0}^{\infty} (-1)^n c_n 7^n$ is divergent ANSWER

(b) $\sum_{n=0}^{\infty} c_n (-5)^n$ is convergent

(c) $\sum_{n=0}^{\infty} (-1)^n c_n 4^n$ is divergent

(d) $\sum_{n=0}^{\infty} c_n 4^n$ is divergent

(e) $\sum_{n=0}^{\infty} c_n 3^n$ is convergent

13. $\int \cos(2x^2)\,dx =$

(a) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+1}}{(4n+1)(2n)!}$ ANSWER

(b) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+4}}{(4n+4)(2n)!}$

(c) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+2}}{(2n+1)(2n)!}$

(d) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+2}}{(2n+1)(2n)!}$

(e) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+3}}{(4n+3)(2n+1)!}$

14. $\sum_{n=0}^{\infty} 5(-6)^n x^{2n} =$

(a) $\frac{-5}{1+6x^2}$, where $|x| < \sqrt{6}$

(b) $\frac{5}{1+6x^2}$, where $|x| < \frac{1}{\sqrt{6}}$ ANSWER

(c) $\frac{5}{(1-6x)^2}$, where $|x| < \sqrt{6}$

(d) $\frac{-5}{1+6x^2}$, where $|x| < \frac{1}{\sqrt{6}}$

(e) $\frac{5}{(1+6x)^2}$, where $|x| < \sqrt{6}$

15. $\sum_{n=1}^{\infty} \frac{\arctan(n)+4}{n^8}$ is:

(a) Convergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{\frac{\pi}{2}+4}{n^8}$ ANSWER

(b) Convergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^8}$

(c) Divergent by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^8}$

(d) Divergent by the Test for Divergence

(e) Divergent by the Integral Test

PART II: Work Out

16. For the power series $\sum_{n=2}^{\infty} \frac{(x+1)^n}{(-3)^n \ln(n)}$:

a.) (4 pts) Find the radius of convergence.

$$\lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{(-3)^{n+1} \ln(n+1)} \cdot \frac{(-3)^n \ln n}{(x+1)^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{x+1}{-3 \ln(n+1)} \cdot \ln n \right|$$

$$\lim_{n\to\infty} \left| \frac{x+1}{-3} \frac{\ln n}{\ln(n+1)} \right|$$

$$\lim_{n\to\infty} \frac{\ln n}{\ln(n+1)} \overset{L}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n+1}}$$

$$= \lim_{n\to\infty} \frac{n+1}{n}$$

$$= 1$$

$$= \left| \frac{x+1}{-3} \right| < 1$$

$$\boxed{R = 3}$$

$$|x+1| < 3$$

b.) (8 pts) Find the interval of convergence. You must test the endpoints for convergence.

$$-3 < x+1 < 3$$

$$-4 < x < 2$$

$x=-4$: $\sum_{n=2}^{\infty} \frac{(-3)^n}{(-3)^n \ln n} = \sum_{n=2}^{\infty} \frac{1}{\ln n} > \sum_{n=2}^{\infty} \frac{1}{n}$

diverges by p-series smaller diverges, so does larger

$x=2$: $\sum_{n=2}^{\infty} \frac{3^n}{(-3)^n \ln n} = \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

converges by AST: $a_n = \frac{1}{\ln n}$ decreases to zero.

$$\boxed{I = (-4, 2]}$$

17. For $f(x) = \frac{2}{x^2}$:

a.) (9 pts) Find the Taylor series at $a = 5$.

$$\sum_{n=0}^{\infty} \frac{f^n(5)}{n!}(x-5)^n$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 2(n+1)!}{n! \cdot 5^{n+2}}(x-5)^n$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{2(-1)^n(n+1)}{5^{n+2}}(x-5)^n}$$

$f = \frac{2}{x^2}$

$f' = \frac{-2 \cdot 2}{x^3}$

$f'' = \frac{3 \cdot 2 \cdot 2}{x^4}$

$f''' = \frac{-4 \cdot 3 \cdot 2 \cdot 2}{x^5}$

$f^n(x) = \frac{(-1)^n 2(n+1)!}{x^{n+2}}$

$f^n(5) = \frac{(-1)^n 2(n+1)!}{5^{n+2}}$

b.) (3 pts) Find the radius of convergence for the Taylor series found in part a.).

RT: $\lim_{n\to\infty} \left| \frac{2(n+2)}{5^{n+3}}(x-5)^{n+1} \cdot \frac{5^{n+2}}{2(n+1)} \right|$

$= \lim_{n\to\infty} \left| \frac{2(n+2)(x-5)}{5} \cdot \frac{1}{2(n+1)} \right|$

$= \lim_{n\to\infty} \left| \frac{x-5}{5} \frac{(n+2)}{n+1} \right| = \frac{|x-5|}{5} < 1$

$|x-5| < 5$

$\boxed{R=5}$

18. (8 pts) Consider $\sum_{n=2}^{\infty} \frac{(-1)^n n^2}{n^3 - 1}$. Determine whether the series converges absolutely, converges but not absolutely, or diverges. Fully support your conclusion.

AST: $a_n = \frac{n^2}{n^3 - 1}$ decreases to zero, so $\sum_{n=2}^{\infty} \frac{(-1)^n n^2}{n^3-1}$ converges.

Test absolute convergence, look at $\sum_{n=2}^{\infty} \frac{n^2}{n^3-1} > \sum_{n=2}^{\infty} \frac{n^2}{n^3} = \sum_{n=2}^{\infty} \frac{1}{n}$ which diverges.

Thus $\sum_{n=2}^{\infty} \frac{(-1)^n n^2}{n^3-1}$ converges, but not absolutely

19. Consider $\sum_{n=1}^{\infty} \frac{1}{(n+1)(\ln(n+1))^2}$.

a.) (5 pts) Prove the series converges.

IT: $\int_1^{\infty} \frac{dx}{(x+1)(\ln(x+1))^2}$

u-sub, where $u = \ln(x+1)$, $du = \frac{dx}{x+1}$

$\int \frac{du}{u^2} = -\frac{1}{u} = -\frac{1}{\ln(x+1)}$

$= \frac{-1}{\ln(x+1)} \Big|_1^{\infty} = -\frac{1}{\infty} + \frac{1}{\ln 2}$

$= \frac{1}{\ln 2} < \infty$ converges

b.) (4 pts) If we used s_3, the third partial sum, to approximate the sum s of the series, use the Remainder Estimate for the Integral Test to find an upper bound on the remainder, $R_3 = s - s_3$.

$R_3 < \int_3^{\infty} \frac{dx}{(x+1)(\ln(x+1))^2} = \frac{-1}{\ln(x+1)} \Big|_3^{\infty}$

$= -\frac{1}{\infty} + \frac{1}{\ln 4}$

$|R_3| < \frac{1}{\ln 4}$

20. (i) (9 pts) Find a power series about zero for $f(x) = \ln(4 + x^2)$.

$$\frac{d}{dx} \ln(4+x^2) = \frac{2x}{4+x^2}$$

$$= \frac{2x}{4(1+\frac{x^2}{4})}$$

$$= \frac{2x}{4} \sum_{n=0}^{\infty} \left(-\frac{x^2}{4}\right)^n, \quad |x| < 2$$

$$= \sum_{n=0}^{\infty} \frac{x}{2} \frac{(-1)^n x^{2n}}{4^n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2} \frac{(-1)^n x^{2n+1}}{4^n}$$

$$\ln(4+x^2) = \int \sum_{n=0}^{\infty} \frac{1}{2} \frac{(-1)^n x^{2n+1}}{4^n} \, dx$$

$$\rightarrow \ln(4+x^2) = C + \sum_{n=0}^{\infty} \frac{\frac{1}{2}(-1)^n x^{2n+2}}{4^n(2n+2)}$$

if $x = 0$,
$C = \ln 4$

$$\ln(4+x^2) = \ln 4 + \sum_{n=0}^{\infty} \frac{\frac{1}{2}(-1)^n x^{2n+2}}{4^n(2n+2)}$$

(ii) (3 pts) What is the radius of convergence of the series above?

$|x| < 2$ from above, so $R = 2$.

(iii) (2 pts) Using (i), find a power series about zero for $g(x) = x^3 \ln(4 + x^2)$.

$$x^3 \ln(4+x^2) = x^3 \left[\ln 4 + \sum_{n=0}^{\infty} \frac{\frac{1}{2}(-1)^n x^{2n+2}}{4^n(2n+2)} \right]$$

$$= x^3 \ln 4 + \sum_{n=0}^{\infty} \frac{\frac{1}{2}(-1)^n x^{2n+5}}{4^n(2n+2)}$$