MATH 152 Spring 2016
COMMON EXAM III - VERSION B

LAST NAME: ______________________ FIRST NAME: ______________________

INSTRUCTOR: ______________________

SECTION NUMBER: __________

UIN: ______________________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-20), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		12
17		12
18		8
19		9
20		14
Total		100

PART I: Multiple Choice. 3 points each.

1. Find the equation of the sphere whose diameter has endpoints $(2, 1, 6)$ and $(4, -3, 8)$.

   (a) $(x+3)^2 + (y-1)^2 + (z+7)^2 = 6$
   (b) $(x-3)^2 + (y+1)^2 + (z-7)^2 = 6$
   (c) $(x-3)^2 + (y+1)^2 + (z-7)^2 = 24$
   (d) $(x-3)^2 + (y-1)^2 + (z-7)^2 = 24$
   (e) $(x-6)^2 + (y+2)^2 + (z-14)^2 = 24$

2. Which of the following statements is true of the series $\sum_{n=1}^{\infty} \frac{n^3}{n^4+n^2+1}$?

   I. Diverges by comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

   II. Diverges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

   III. Diverges by Test for Divergence

   (a) Only I and II are true
   (b) Only I is true
   (c) Only II is true
   (d) Only III is true
   (e) None of I, II and III is true.

3. Suppose it is known that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -3$ and diverges when $x = 5$. Which of the following is certain to be true?

   (a) $\sum_{n=0}^{\infty} c_n 3^n$ is convergent
   (b) $\sum_{n=0}^{\infty} c_n (-5)^n$ is convergent
   (c) $\sum_{n=0}^{\infty} (-1)^n c_n 4^n$ is divergent
   (d) $\sum_{n=0}^{\infty} c_n 4^n$ is divergent
   (e) $\sum_{n=0}^{\infty} (-1)^n c_n 7^n$ is divergent

4. $\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{2\pi}{3}\right)^{2n+1}}{(2n+1)!} =$

(a) $-\frac{1}{2}$

(b) $\frac{\sqrt{3}}{2}$

(c) $-\frac{\sqrt{3}}{2}$

(d) $\frac{1}{2}$

(e) $e^{2\pi/3}$

5. Write $f(x) = \frac{x^3}{8+x}$ as a power series centered at 0.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3}}{8^{n+1}}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3}}{64^n}$

(c) $\sum_{n=0}^{\infty} \left(-\frac{x}{8}\right)^{n+3}$

(d) $\sum_{n=0}^{\infty} \frac{x^{n+3}}{8^{n+1}}$

(e) $\sum_{n=0}^{\infty} \left(-\frac{x}{8}\right)^{3n}$

6. Find the third degree Taylor Polynomial, $T_3(x)$, for $f(x) = \cos(2x)$ at $a = \frac{\pi}{6}$.

(a) $T_3(x) = \frac{1}{2} - \sqrt{3}\left(x - \frac{\pi}{6}\right) - 2\left(x - \frac{\pi}{6}\right)^2 + 4\sqrt{3}\left(x - \frac{\pi}{6}\right)^3$

(b) $T_3(x) = \frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3$

(c) $T_3(x) = \frac{\sqrt{3}}{2} - \left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right)^3$

(d) $T_3(x) = \frac{1}{2} - \sqrt{3}\left(x - \frac{\pi}{6}\right) - \left(x - \frac{\pi}{6}\right)^2 + \frac{2\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right)^3$

(e) $T_3(x) = \frac{1}{2} - \sqrt{3}\left(x - \frac{\pi}{6}\right) + \left(x - \frac{\pi}{6}\right)^2 + \frac{2\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right)^3$

7. The region in $\mathbb{R}^3$ described by the equation $y = 8$ is:

(a) A line parallel to the x-axis.
(b) A plane parallel to the xz plane passing through the point $(0, 8, 0)$.
(c) The point $(0, 8, 0)$.
(d) A plane parallel to the yz plane passing through the point $(0, 8, 0)$.
(e) A line parallel to the z-axis.

8. Using the Alternating Series Estimation Theorem, what is the smallest number of terms we must use to approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ with error less than $\frac{1}{120}$?

(a) $n = 11$
(b) $n = 9$
(c) $n = 10$
(d) $n = 12$
(e) $n = 13$

9. Find the radius of convergence, R, and the interval of convergence, I, for $\sum_{n=1}^{\infty} \frac{(x+5)^n n!}{3^n}$.

(a) $R = 3$, $I = (-8, -2)$
(b) $R = \infty$, $I = (-\infty, \infty)$
(c) $R = \infty$, $I = \{-5\}$
(d) $R = 3$, $I = (2, 8)$
(e) $R = 0$, $I = \{-5\}$

10. For which of the following series is the Ratio Test inconclusive?

I) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+3}$

II) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

III) $\sum_{n=1}^{\infty} n4^n$

(a) I and II only
(b) II only
(c) I only
(d) II and III only
(e) I, II, and III

11. Which of the following is a Maclaurin series for $f(x) = x^3 e^{-x^2}$?

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n)!}$

(b) $\sum_{n=0}^{\infty} \frac{x^{2n+3}}{n!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{n!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{5n}}{n!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{n!}$

12. $\sum_{n=1}^{\infty} \frac{\arctan(n)+4}{n^8}$ is:

(a) Convergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^8}$

(b) Convergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{\frac{\pi}{2}+4}{n^8}$

(c) Divergent by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^8}$

(d) Divergent by the Test for Divergence

(e) Divergent by the Integral Test

13. $\int \cos(2x^2)\,dx =$

(a) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+3}}{(4n+3)(2n+1)!}$

(b) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+4}}{(4n+4)(2n)!}$

(c) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+2}}{(2n+1)(2n)!}$

(d) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+2}}{(2n+1)(2n)!}$

(e) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+1}}{(4n+1)(2n)!}$

14. $\sum_{n=0}^{\infty} 5(-6)^n x^{2n} =$

(a) $\frac{5}{1+6x^2}$, where $|x| < \frac{1}{\sqrt{6}}$

(b) $\frac{-5}{1+6x^2}$, where $|x| < \sqrt{6}$

(c) $\frac{5}{(1-6x)^2}$, where $|x| < \sqrt{6}$

(d) $\frac{-5}{1+6x^2}$, where $|x| < \frac{1}{\sqrt{6}}$

(e) $\frac{5}{(1+6x)^2}$, where $|x| < \sqrt{6}$

15. Find $f^{(23)}(3)$, that is, the 23^{rd} derivative of $f(x)$ at $x = 3$, if $f(x) = \sum_{n=0}^{\infty} \frac{(-3)^{n+2}}{(2n)!}(x-3)^n$.

(a) $\frac{(-3)^{24}(23)!}{(46)!}$

(b) $\frac{(-3)^{25}(24)!}{(46)!}$

(c) $\frac{(-3)^{25}}{(46)!}$

(d) $\frac{(-3)^{25}(23)!}{(46)!}$

(e) 0

PART II: Work Out

16. For the power series $\sum_{n=2}^{\infty} \frac{(x+1)^n}{(-5)^n \ln(n)}$:

a.) (4 pts) Find the radius of convergence.

b.) (8 pts) Find the interval of convergence. You must test the endpoints for convergence.

17. For $f(x) = \dfrac{2}{x^2}$:

a.) (9 pts) Find the Taylor series at $a = 3$.

b.) (3 pts) Find the radius of convergence for the Taylor series found in part a.).

18. (8 pts) Consider $\sum_{n=2}^{\infty} \frac{(-1)^n n^3}{n^4 - 1}$. Determine whether the series converges absolutely, converges but not absolutely, or diverges. Fully support your conclusion.

19. Consider $\sum_{n=1}^{\infty} \frac{1}{(n+2)(\ln(n+2))^2}$.

a.) (5 pts) Prove the series converges.

b.) (4 pts) If we used s_3, the third partial sum, to approximate the sum s of the series, use the Remainder Estimate for the Integral Test to find an upper bound on the remainder, $R_3 = s - s_3$.

20. (i) (9 pts) Find a power series about zero for $f(x) = \ln(9 + x^2)$.

(ii) (3 pts) What is the radius of convergence of the series above?

(iii) (2 pts) Using (i), find a power series about zero for $g(x) = x^3 \ln(9 + x^2)$.