MATH 152 Spring 2016
COMMON EXAM III - VERSION B

LAST NAME: Answers FIRST NAME: ______________

INSTRUCTOR: ______________

SECTION NUMBER: ______________

UIN: ______________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 3 points.
4. In Part 2 (Problems 16-20), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-15		45
16		12
17		12
18		8
19		9
20		14
Total		100

PART I: Multiple Choice. 3 points each.

1. Find the equation of the sphere whose diameter has endpoints $(2, 1, 6)$ and $(4, -3, 8)$.

   (a) $(x+3)^2 + (y-1)^2 + (z+7)^2 = 6$
   (b) $(x-3)^2 + (y+1)^2 + (z-7)^2 = 6$ ANSWER
   (c) $(x-3)^2 + (y+1)^2 + (z-7)^2 = 24$
   (d) $(x-3)^2 + (y-1)^2 + (z-7)^2 = 24$
   (e) $(x-6)^2 + (y+2)^2 + (z-14)^2 = 24$

2. Which of the following statements is true of the series $\sum_{n=1}^{\infty} \frac{n^3}{n^4+n^2+1}$?

   I. Diverges by comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

   II. Diverges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

   III. Diverges by Test for Divergence

   (a) Only I and II are true
   (b) Only I is true
   (c) Only II is true ANSWER
   (d) Only III is true
   (e) None of I, II and III is true.

3. Suppose it is known that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -3$ and diverges when $x = 5$. Which of the following is certain to be true?

   (a) $\sum_{n=0}^{\infty} c_n 3^n$ is convergent

   (b) $\sum_{n=0}^{\infty} c_n (-5)^n$ is convergent

   (c) $\sum_{n=0}^{\infty} (-1)^n c_n 4^n$ is divergent

   (d) $\sum_{n=0}^{\infty} c_n 4^n$ is divergent

   (e) $\sum_{n=0}^{\infty} (-1)^n c_n 7^n$ is divergent ANSWER

4. $\sum_{n=0}^{\infty} \dfrac{(-1)^n \left(\frac{2\pi}{3}\right)^{2n+1}}{(2n+1)!} =$

(a) $-\dfrac{1}{2}$

(b) $\dfrac{\sqrt{3}}{2}$ ANSWER

(c) $-\dfrac{\sqrt{3}}{2}$

(d) $\dfrac{1}{2}$

(e) $e^{2\pi/3}$

5. Write $f(x) = \dfrac{x^3}{8+x}$ as a power series centered at 0.

(a) $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^{n+3}}{8^{n+1}}$ ANSWER

(b) $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^{n+3}}{64^n}$

(c) $\sum_{n=0}^{\infty} \left(-\dfrac{x}{8}\right)^{n+3}$

(d) $\sum_{n=0}^{\infty} \dfrac{x^{n+3}}{8^{n+1}}$

(e) $\sum_{n=0}^{\infty} \left(-\dfrac{x}{8}\right)^{3n}$

6. Find the third degree Taylor Polynomial, $T_3(x)$, for $f(x) = \cos(2x)$ at $a = \dfrac{\pi}{6}$.

(a) $T_3(x) = \dfrac{1}{2} - \sqrt{3}\left(x - \dfrac{\pi}{6}\right) - 2\left(x - \dfrac{\pi}{6}\right)^2 + 4\sqrt{3}\left(x - \dfrac{\pi}{6}\right)^3$

(b) $T_3(x) = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\left(x - \dfrac{\pi}{6}\right) - \dfrac{1}{4}\left(x - \dfrac{\pi}{6}\right)^2 + \dfrac{\sqrt{3}}{12}\left(x - \dfrac{\pi}{6}\right)^3$

(c) $T_3(x) = \dfrac{\sqrt{3}}{2} - \left(x - \dfrac{\pi}{6}\right) - \dfrac{\sqrt{3}}{4}\left(x - \dfrac{\pi}{6}\right)^2 + \dfrac{\sqrt{3}}{3}\left(x - \dfrac{\pi}{6}\right)^3$

(d) $T_3(x) = \dfrac{1}{2} - \sqrt{3}\left(x - \dfrac{\pi}{6}\right) - \left(x - \dfrac{\pi}{6}\right)^2 + \dfrac{2\sqrt{3}}{3}\left(x - \dfrac{\pi}{6}\right)^3$ ANSWER

(e) $T_3(x) = \dfrac{1}{2} - \sqrt{3}\left(x - \dfrac{\pi}{6}\right) + \left(x - \dfrac{\pi}{6}\right)^2 + \dfrac{2\sqrt{3}}{3}\left(x - \dfrac{\pi}{6}\right)^3$

7. The region in $\mathbb{R}^3$ described by the equation $y = 8$ is:

(a) A line parallel to the x-axis.

(b) A plane parallel to the xz plane passing through the point $(0, 8, 0)$. ANSWER

(c) The point $(0, 8, 0)$.

(d) A plane parallel to the yz plane passing through the point $(0, 8, 0)$.

(e) A line parallel to the z-axis.

8. Using the Alternating Series Estimation Theorem, what is the smallest number of terms we must use to approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ with error less than $\frac{1}{120}$?

(a) $n = 11$

(b) $n = 9$

(c) $n = 10$ ANSWER

(d) $n = 12$

(e) $n = 13$

9. Find the radius of convergence, R, and the interval of convergence, I, for $\sum_{n=1}^{\infty} \frac{(x+5)^n n!}{3^n}$.

(a) $R = 3$, $I = (-8, -2)$

(b) $R = \infty$, $I = (-\infty, \infty)$

(c) $R = \infty$, $I = \{-5\}$

(d) $R = 3$, $I = (2, 8)$

(e) $R = 0$, $I = \{-5\}$ ANSWER

10. For which of the following series is the Ratio Test inconclusive?

I) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+3}$

II) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

III) $\sum_{n=1}^{\infty} n4^n$

(a) I and II only CORRECT

(b) II only

(c) I only

(d) II and III only

(e) I, II, and III

11. Which of the following is a Maclaurin series for $f(x) = x^3 e^{-x^2}$?

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n)!}$

(b) $\sum_{n=0}^{\infty} \frac{x^{2n+3}}{n!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{n!}$ ANSWER

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{5n}}{n!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{n!}$

12. $\sum_{n=1}^{\infty} \frac{\arctan(n)+4}{n^8}$ is:

(a) Convergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^8}$

(b) Convergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{\frac{\pi}{2}+4}{n^8}$ ANSWER

(c) Divergent by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^8}$

(d) Divergent by the Test for Divergence

(e) Divergent by the Integral Test

13. $\int \cos(2x^2)\,dx =$

(a) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+3}}{(4n+3)(2n+1)!}$

(b) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+4}}{(4n+4)(2n)!}$

(c) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+2}}{(2n+1)(2n)!}$

(d) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+2}}{(2n+1)(2n)!}$

(e) $C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+1}}{(4n+1)(2n)!}$ ANSWER

14. $\sum_{n=0}^{\infty} 5(-6)^n x^{2n} =$

(a) $\frac{5}{1+6x^2}$, where $|x| < \frac{1}{\sqrt{6}}$ ANSWER

(b) $\frac{-5}{1+6x^2}$, where $|x| < \sqrt{6}$

(c) $\frac{5}{(1-6x)^2}$, where $|x| < \sqrt{6}$

(d) $\frac{-5}{1+6x^2}$, where $|x| < \frac{1}{\sqrt{6}}$

(e) $\frac{5}{(1+6x)^2}$, where $|x| < \sqrt{6}$

15. Find $f^{(23)}(3)$, that is, the 23^{rd} derivative of $f(x)$ at $x = 3$, if $f(x) = \sum_{n=0}^{\infty} \frac{(-3)^{n+2}}{(2n)!}(x-3)^n$.

(a) $\frac{(-3)^{24}(23)!}{(46)!}$

(b) $\frac{(-3)^{25}(24)!}{(46)!}$

(c) $\frac{(-3)^{25}}{(46)!}$

(d) $\frac{(-3)^{25}(23)!}{(46)!}$ ANSWER

(e) 0

PART II: Work Out

16. For the power series $\sum_{n=2}^{\infty} \frac{(x+1)^n}{(-5)^n \ln(n)}$:

a.) (4 pts) Find the radius of convergence.

$$\lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{(-5)^{n+1}\ln(n+1)} \cdot \frac{(-5)^n \ln n}{(x+1)^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{x+1}{\ln(n+1)} \frac{(\ln n)}{-5} \right|$$

$$\lim_{n\to\infty} \frac{\ln n}{\ln(n+1)} \overset{L}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n+1}}$$

$$\lim_{n\to\infty} \frac{n+1}{n} = 1$$

$$= \left| \frac{x+1}{-5} \right| < 1$$

$$|x+1| < 5$$

$$\boxed{R = 5}$$

b.) (8 pts) Find the interval of convergence. You must test the endpoints for convergence.

$$|x+1| < 5$$

$$-5 < x+1 < 5$$

$$-6 < x < 4$$

$$I = (-6, 4]$$

$x=-6$: $\sum_{n=2}^{\infty} \frac{(-5)^n}{(-5)^n \ln n} = \sum_{n=2}^{\infty} \frac{1}{\ln n}$

$\sum_{n=2}^{\infty} \frac{1}{\ln n} > \sum_{n=2}^{\infty} \frac{1}{n}$ diverges by C.T.

$x=4$: $\sum_{n=2}^{\infty} \frac{5^n}{(-5)^n \ln n} = \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

converges by AST since $\frac{1}{\ln n}$ decreases to 0.

17. For $f(x) = \frac{2}{x^2}$:

a.) (9 pts) Find the Taylor series at $a = 3$.

$$\sum_{n=0}^{\infty} \frac{f^n(3)}{n!}(x-3)^n \qquad f(x) = \frac{2}{x^2}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 2(n+1)!}{n!\, 3^{n+2}}(x-3)^n \qquad f'(x) = \frac{-2\cdot 2}{x^3}$$

$$f'' = \frac{-2\cdot 3\cdot 2}{x^4}$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n (n+1)(2)(x-3)^n}{3^{n+2}}} \qquad f''' = \frac{2\cdot 4\cdot 3\cdot 2}{x^5}$$

$$f^n(x) = \frac{(-1)^n 2(n+1)!}{x^{n+2}}$$

$$f^n(3) = \frac{(-1)^n 2(n+1)!}{3^{n+2}}$$

b.) (3 pts) Find the radius of convergence for the Taylor series found in part a.).

$$\lim_{n\to\infty} \left| \frac{(n+2)(\cancel{2})(x-3)^{n+1}}{3^{n+3}} \cdot \frac{3^{n+2}}{(n+1)(\cancel{2})(x-3)^n} \right|$$

$$= \left| \frac{x-3}{3} \right| < 1$$

$$|x-3| < 3 \qquad \boxed{R = 3}$$

18. (8 pts) Consider $\sum_{n=2}^{\infty} \frac{(-1)^n n^3}{n^4-1}$. Determine whether the series converges absolutely, converges but not absolutely, or diverges. Fully support your conclusion.

AST: $a_n = \frac{n^3}{n^4-1}$ which decreases to zero, so $\sum_{n=2}^{\infty} \frac{(-1)^n n^3}{n^4-1}$ converges by AST

For absolute convergence, $\sum_{n=2}^{\infty} \frac{n^3}{n^4-1} > \sum_{n=2}^{\infty} \frac{n^3}{n^4} = \sum_{n=2}^{\infty} \frac{1}{n}$ which diverges.

Thus $\sum_{n=2}^{\infty} \frac{(-1)^n n^3}{n^4-1}$ converges but not absolutely.

19. Consider $\sum_{n=1}^{\infty} \frac{1}{(n+2)(\ln(n+2))^2}$.

a.) (5 pts) Prove the series converges.

integral test:

$$\int_1^{\infty} \frac{dx}{(x+2)(\ln(x+2))^2} = -\frac{1}{\ln(x+2)}\Big|_1^{\infty}$$

$$= -\frac{1}{\infty} + \frac{1}{\ln 3} < \infty$$

series converges.

u-sub
$u = \ln(x+2)$ $\quad du = \frac{dx}{x+1}$

$$\int \frac{du}{u^2} = -\frac{1}{u} = \frac{-1}{\ln(x+2)}$$

b.) (4 pts) If we used s_3, the third partial sum, to approximate the sum s of the series, use the Remainder Estimate for the Integral Test to find an upper bound on the remainder, $R_3 = s - s_3$.

$$R_3 < \int_3^{\infty} \frac{dx}{(x+2)(\ln(x+2))^2}$$

$$= -\frac{1}{\ln(x+2)}\Big|_3^{\infty}$$

$$= -\frac{1}{\infty} + \frac{1}{\ln 5}$$

so $\boxed{R_3 < \frac{1}{\ln 5}}$

20. (i) (9 pts) Find a power series about zero for $f(x) = \ln(9 + x^2)$.

$$\frac{d}{dx} \ln(9+x^2) = \frac{2x}{9+x^2}$$

$$= \frac{2x}{9(1+\frac{x^2}{9})}$$

$$= \frac{2x}{9} \sum_{n=0}^{\infty} \left(-\frac{x^2}{9}\right)^n, \text{ where } |x| < 3$$

$$= \frac{2x}{9} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{9^n}$$

$$= 2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{9^{n+1}}$$

$$\ln(9+x^2) = \int 2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{9^{n+1}} \, dx$$

$$= C + 2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+2)9^{n+1}}$$

where $C = \ln 9$

$$\ln(9+x^2) = \ln 9 + 2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+2)9^{n+1}}$$

(ii) (3 pts) What is the radius of convergence of the series above?

$|R| < 3$ from above, so $R = 3$

(iii) (2 pts) Using (i), find a power series about zero for $g(x) = x^3 \ln(9 + x^2)$.

$$x^3 \ln(9+x^2) = x^3 \left[\ln 9 + 2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+2)9^{n+1}} \right]$$

$$= x^3 \ln 9 + 2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+5}}{(2n+2)(9^{n+1}}$$