MATH 152

Exam 2

Fall 1997

Version A

Student (Print) _____
Last, First Middle

Student (Sign) _____

Student ID _____

Instructor _____

Section _____

1-10	
11	
12	
13	
14	
15	
TOTAL	

Part I is multiple choice. There is no partial credit. You may not use a calculator.

Part II is work out. Show all your work. Partial credit will be given. You may use your calculator.

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator. You have 1 hour.

1. The natural length of a certain spring is at $2\,\mathrm{m}$. A force of $6\,\mathrm{N}$ will stretch the spring from $2\,\mathrm{m}$ to $4\,\mathrm{m}$. How much work will it take to stretch the spring from $4\,\mathrm{m}$ to $6\,\mathrm{m}$?

a. $18\,\mathrm{J}$

b. $24\,\mathrm{J}$

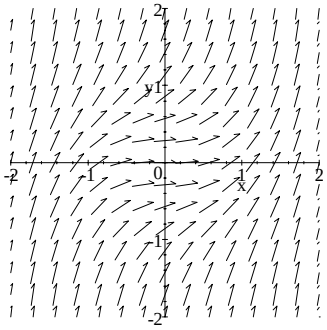
c. $36\,\mathrm{J}$

d. $48\,\mathrm{J}$

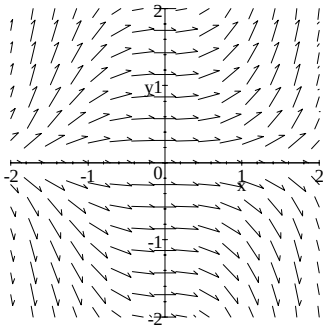
e. $60\,\mathrm{J}$

2. The differential equation $\frac{dy}{dx} = x^2 - y^2$ has direction field

a.



b.



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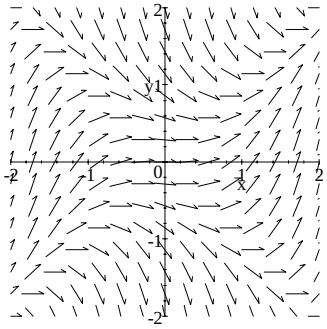

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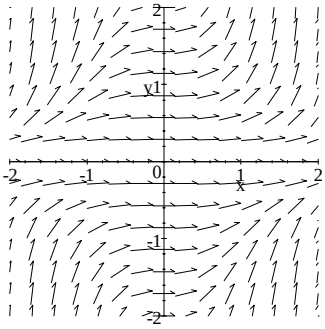
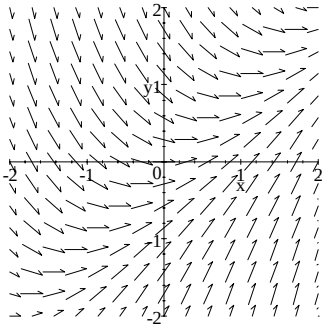


e.



3. The differential equation $\frac{dy}{dx} = (2x+1)(y+1)$ is

a. neither separable nor linear
b. separable but not linear
c. linear but not separable
d. separable and linear
e. none of these

4. Solve the differential equation $\frac{dy}{dx} = \frac{3}{4}\frac{x^2}{y^3}$ with the initial condition $y(0) = 2$.

a. $x^{3/4} + 2$
b. $\sqrt[3]{x^4 + 8}$
c. $x^{4/3} + 2$
d. $2x^{4/3}$
e. $\sqrt[4]{x^3 + 16}$

5. If $\mathbf{a} = \langle 4, 2, 1\rangle$ and $\mathbf{b} = \langle 3, 2, 3\rangle$ then $\mathbf{a} + 2\mathbf{b} =$

a. $\langle -2, -2, -5\rangle$
b. $\langle 10, 6, 7\rangle$
c. $\langle 7, 4, 4\rangle$
d. $\langle 1, 0, -2\rangle$
e. $\langle 11, 6, 5\rangle$

6. The triangle with vertices $P = (4, 2, 1)$, $Q = (3, 3, 1)$ and $R = (3, 2, 3)$ is

a. equilateral
b. isosceles but not right
c. right but not isosceles
d. isosceles and right
e. none of these

7. In 3-dimensional space the equation $x^2 + z^2 < 16$ describes

a. the interior of a circle of radius 4 in the xz-plane centered at the origin.

b. the region inside a paraboloid centered along the positive y-axis.

c. the region inside a cone centered along the positive y-axis.

d. the part of the xz-plane outside of a circle of radius 4 centered at the origin.

e. the interior of a cylinder of radius 4 centered on the y-axis.

8. The arclength of the curve $y = e^{2x}$ between $x = 1$ and $x = 2$ is given by

a. $\int_1^2 \sqrt{1 + e^{4x}}\, dx$

b. $\int_{e^2}^{e^4} \sqrt{1 + 2e^{2x}}\, dx$

c. $\int_1^2 \sqrt{1 + 4e^{4x}}\, dx$

d. $\int_{e^2}^{e^4} 2\pi \sqrt{1 + e^{2x}}\, dx$

e. $\int_1^2 2\pi x \sqrt{1 + e^{4x}}\, dx$

9. The curve $y = \sin(x)$, between $x = 1$ and $x = \frac{\pi}{2}$, is rotated about the x-axis. The surface area of the resulting surface is given by.

a. $\int_1^{\pi/2} 2\pi \sin(x) \sqrt{1 + \cos^2(x)}\ dx$

b. $\int_1^{\pi/2} 2\pi x \sqrt{1 + \cos^2(x)}\ dx$

c. $\int_1^{\pi/2} 2\pi \cos(x) \sqrt{1 + \sin^2(x)}\ dx$

d. $\int_0^1 2\pi y \sqrt{\cos^{-2}(y) + 1}\ dy$

e. $\int_1^{\pi/2} 2\pi x \sqrt{1 + \sin^2(x)}\ dx$

10. Find a unit vector in the direction of the vector $(3, -4, 12)$

a. $\left(\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}\right)$

b. $\left(-\frac{12}{13}, \frac{4}{13}, -\frac{3}{13}\right)$

c. $\left(-\frac{3}{13}, \frac{4}{13}, -\frac{12}{13}\right)$

d. $\left(\frac{3}{13}, -\frac{4}{13}, \frac{12}{13}\right)$

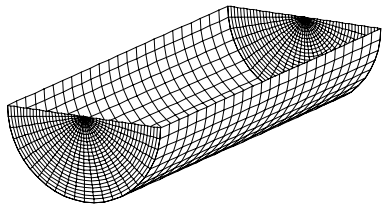
e. $\left(-\frac{4}{13}, \frac{12}{13}, \frac{4}{13}\right)$

Part II: Work Out (10 points each)
Show all your work. Partial credit will be given.
You may use your calculator but only after 1 hour.

11. A plate in the shape of an isosceles triangle with base of $6\,\mathrm{m}$ and altitude of $9\,\mathrm{m}$ is put in water with the base at the surface of the water and the point straight down. Find the total force on the plate. Note the density of water is $1000\,\mathrm{kg/m^3}$ and the acceleration of gravity is $9.8\,\mathrm{m/s^2}$.

12. Solve the differential equation $\frac{dy}{dx} = 3x^2y + 6x^2e^{x^3}$ with the initial condition $y(0) = 5$.

13. A water trough has the shape of half of a cylinder on its side, as shown in the figure. The radius is $2\,\mathrm{m}$ and the length is $10\,\mathrm{m}$. How much work does it take to pump the water out of the top of the trough if the trough is full of water? The density of water is $1000\,\mathrm{kg/m^3}$.



14. Find the x-coordinate of the centroid of the area bounded by the parabola $y = x^2$ and the lines $x = 1$ and $y = 0$.

15. A barrel initially contains 3 cups of sugar dissolved in 4 gallons of water. You then add pure water at the rate of 2 gallons per minute while the mixture is draining out of a hole in the bottom at 2 gallons per minute. Find the amount of sugar in the barrel after 1 minute.

Note: You must explicitly show the differential equation, the initial condition, the method of solution, the general solution, the solution satisfying the initial condition and the final answer.